Solutions 11

1. a) The posterior distribution is from Bayes rule,

$$\begin{aligned} p(\theta \mid y_1, \dots, y_N) &= \frac{\mathbb{P}(y_1, \dots, y_N \mid \theta) p_0(\theta)}{\int_{\mathbb{R}} d\theta \, \mathbb{P}(y_1, \dots, y_N \mid \theta) p_0(\theta)} \\ &= \frac{p_0(\theta) \prod_{i=1}^N q(y_i \mid \theta)}{\int_{\mathbb{R}} d\theta p_0(\theta) \prod_{i=1}^N q(y_i \mid \theta)} \end{aligned}$$

The base (or proposal) chain has probabilities $\psi_{\theta^t \to \theta^{t+1}} = p_0(\theta^{t+1})$ thus

$$\begin{aligned} \frac{p(\theta^{t+1} \mid y_1, \dots, y_N) \psi_{\theta^{t+1} \to \theta^t}}{p(\theta^t \mid y_1, \dots, y_N) \psi_{\theta^t \to \theta^{t+1}}} &= \frac{p(\theta^{t+1} \mid y_1, \dots, y_N) p_0(\theta^t)}{p(\theta^t \mid y_1, \dots, y_N) p_0(\theta^{t+1})} \\ &= \frac{\prod_{i=1}^N q(y_i \mid \theta^{t+1})}{\prod_{i=1}^N q(y_i \mid \theta^t)} \end{aligned}$$

So the acceptance probabilities are simply:

$$A_{\theta^t \to \theta^{t+1}} = \min\left\{1, \frac{\prod_{i=1}^N q(y_i \mid \theta^{t+1})}{\prod_{i=1}^N q(y_i \mid \theta^t)}\right\}$$

b) Although the acceptance probabilities do not use the prior we still need to sample from the prior to *propose* a move. However this is in principle much simple than sampling directly from the posterior (whose formula is above).