SCHOOL OF ENGINEERING
MECHANICAL ENGINEERING
LRESE - Laboratory of Renewable Energy Sciences and Engineering

Renewable Energy: Thermodynamics Solution

In this exercise, you will evaluate thermodynamic power cycles and apply the thermodynamic basics. One cycle is relevant for combustion processes (combined Rankine and Brayton) and one is relevant for geothermal processes (combined flash and ORC cycle).

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1. The following assumptions can be used throughout the exercise:

   - Steam generation, heat exchange and condensing occur isobarically
   - Turbines, pump and compressor operate without heat losses
   - Heat exchangers operate adiabatically and isobarically
   - Electricity generators have an efficiency of 100%

   Hint: The information given in a) remains valid in b). b) can be solved without knowledge of the solution of a).

   (a) Consider the gas turbine (system 1) only. Assume that air and exhaust gases behaves as a perfect gas with $\kappa = c_p/c_v = 1.3$ and specific heat of $c_p = 1100$ J/kg/K (κ and c_p are constant). Neglect the addition of mass by added fuel. The energy added by the combustion of fuel is taken into account by $\dot{Q}_{\text{combustion}}$. A mass flow of air of $\dot{m}_{\text{air}} = 20$ kg/s enters the compressor at environment conditions ($p_a = 1$bar, $T_a = 300$K). The compressor operates isentropically, the combustion occurs isobarically, and the expansion in the turbine is isentropic. At compressor outlet the air pressure reaches $p_b = 10$bar. The maximum temperature reached is 1500K. After expansion the exhaust temperature is $T_d = 700$K.

      (i) Draw the pV and Ts-diagrams of the processes $a - b - c - d$.

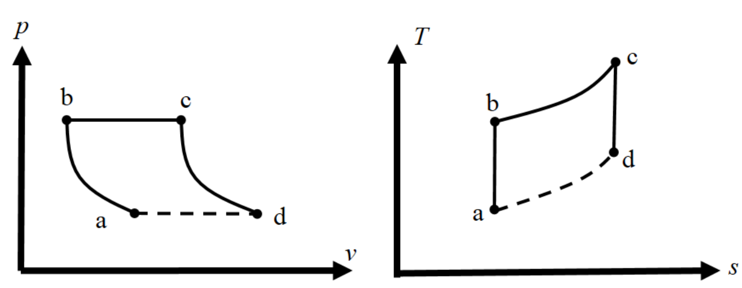

Figure 1: P-V diagram

(ii) Calculate the molar mass of air. What is the volumetric flow rate in m^3/s ($\dot{V}_{\text{air}}$) at the compressor inlet?
Solution:
$\tilde{R}/M = c_p \frac{\kappa-1}{\kappa} \rightarrow M = \tilde{R}/(c_p \frac{\kappa-1}{\kappa}) = 0.032$ kg/mol
$\dot{V} = \frac{\dot{m}\tilde{R}/M \cdot T}{p} = 15.23\ \text{m}^3/\text{s}$

(iii) What is the temperature T_b?
Solution:
$pv^\kappa = const.\ p(\frac{\tilde{R}}{M}\frac{T}{p})^\kappa = \text{const} \rightarrow \left(\frac{p_b}{p_a}\right)^{(\kappa-1)/\kappa} T_a = T_b = 510.4$ K

(iv) How much energy $\dot{Q}_{\text{combustion}}$ is added during combustion?
Solution:
$\dot{Q}_{in} = \dot{m}c_p\Delta T = 21.772$ MW

(v) What is the thermal efficiency of system 1 ($\eta_{th,1}$)?
Hint: From the point of view of system 1, $\dot{Q}_{\text{e1}}$ is completely lost.
Solution:
$\eta = \frac{\dot{W}_{turb}+\dot{W}_{comp}}{\dot{Q}_{in}} = \frac{\dot{m}c_p(T_c-T_d-T_b+T_a)}{\dot{Q}_{in}} = 0.596$

(b) Consider the steam turbine (system 2) only. After the pump the working medium is liquid water at $p_1 = 80$bar and with a specific enthalpy of $h_1 = 1300$ kJ/kg and a mass flux of $\dot{m}_{\text{water}} = 5$ kg/s. Due to the added heat ($\dot{Q}_{\text{he}} + \dot{Q}_{\text{sg}}$) the state of superheated steam is reached before entering the turbine ($p_3 = 80$bar, $T_3 = 510$°C). The isentropic efficiency of the steam turbine is given as $\eta_{\text{is,steam turbine}} = 1$ and the turbine exit pressure is $p_4 = 0.06$bar. After the condenser the state of saturated liquid water is reached. The properties of water may be obtained from http://webbook.nist.gov/chemistry/fluid/.

(i) How much heat is added to the steam in total ($\dot{Q}_{\text{he}} + \dot{Q}_{\text{sg}}$)?
Solution:
h_3 (read from NIST data) $= 3424$ kJ/kg
$\dot{Q}_{tot} = \dot{Q}_{he} + \dot{Q}_{sg} = \dot{m}(h_3 - h_1) = 10.62$ MW

(ii) Assuming that the exhaust gases of system 1 are cooled down from $T_d = 700K$ to $T_e = 400K$, what is the percentage of heat recovered from the gas turbine exhaust ($\frac{\dot{Q}_{\text{he}}}{\dot{Q}_{\text{he}}+\dot{Q}_{\text{sg}}}$)?
Hint: For properties of air refer to the information given in a).
Solution:
$\dot{Q}_{he} = \dot{m}_{air}(h_d - h_e) = \dot{m}_{air}c_p(T_d - T_e) = 6.6$ MW
$\frac{\dot{Q}_{he}}{\dot{Q}_{tot}} = 0.6215$

(iii) What is the state of the working fluid at point 2? Calculate x_2.
Solution:
The fluid is in the two-phase region
$h_2 = h_1 + \frac{\dot{Q}_{he}}{\dot{m}} = 2620$ kJ/kg

$p_2 = p_1$
$x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{2620 - 1317.3}{2758.7 - 1317.3} = 0.9038$

(iv) What is the pump work? Do not use any approximation regarding the incompressibility of water.
Solution:
$p_5 = p_4 =$ 0.06 bar and saturated liquid $\rightarrow h_5 = h_f = 151.48$ kJ/kg
$\dot{W}_{pump} = \dot{m}(h_5 - h_1) =$ -5742.6 kW

(v) What is the thermal efficiency of the steam cycle ($\eta_{th,2}$)?
Hint: From the point of view of system 2 $\dot{Q}_{he}$ is considered as input.
Solution:
$s_3 = 6.7583$ kJ/kg/K
$s_4 = s_3$
$x_4 = \frac{s_4 - s_f}{s_g - s_f} = \frac{6.7583 - 0.521}{8.3290 - 0.521} = 0.799$
$h_4 = h_f + x_4(h_v - h_f) = 151.48 + x_4(2566.6 - 151.48) = 2080.76$ kJ/kg
$\dot{W}_{turb} = \dot{m}(h_3 - h_4) = 6.716$ MW
$\eta = \frac{\dot{W}_{turb}}{\dot{Q}_{tot} - \dot{W}_{pump}} = 0.4105$

(c) What is the thermal efficiency $\eta_{th,3}$ of the combined cycle (system 3)?
Solution:
$\dot{Q}_{sg} = \dot{Q}_{tot} - \dot{Q}_{he} = 4.02$ MW
$\eta = \frac{\dot{W}_{net} + \dot{W}_{turb}}{\dot{Q}_{sg} + \dot{Q}_{comb} - \dot{W}_{pump}}$
$\dot{W}_{net}$ from system 1 $= \dot{m}c_p(T_c - T_d - T_b + T_a) =$ 20 * 1100 * (1500-700 -510.4+300) = 12.97 MW
$\eta = 0.624$

2. A geothermal resource exists as saturated liquid at 230 °C. The geothermal liquid (assumed to be water) is withdrawn from the production well at a rate of 230 kg/s, and is flashed to a pressure of 500 kPa by an isenthalpic flashing process where the resulting vapor is separated from the liquid in a separator and directed to the turbine. The steam leaves the turbine at 10 kPa with a moisture content of 10 percent and enters the condenser where it is condensed and routed to a reinjection well along with liquid coming off the separator and heat exchanger.
For the bottoming cycle, geothermal liquid water leaves the heat exchanger at 90 °C while isobutane enters the isobutane turbine at 3.25 MPa and 145 °C and leaves it at 80 °C and 400 kPa. Isobutane is condensed in an air-cooled condenser and then pumped to the heat exchanger pressure. Assume an isentropic efficiency of 90 % for the pump. The properties of water and isobutane may be obtained from http://webbook.nist.gov/chemistry/fluid/.

Draw the topping and bottoming cycle in a Ts-diagram. Determine

(a) the mass flow rate of steam through the steam turbine.
Solution:
h_1 (from NIST database) = 990.19 kJ/kg
$h_1 = h_2$
p_2 = 500 kPa i.e. 5 bar

$x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{990.19 - 640.09}{2748.1 - 640.09} = 0.166$
Hence $\dot{m}_g$ = 0.166 * 230 = 38.2 kg/s; and $\dot{m}_l$ = (1-0.166) * 230 = 191.8 kg/s

(b) the isentropic efficiency of the steam turbine.
Solution
$p_3 = p_2$
h_3 = 2748.1 kJ/kg (for gas at 3 state)
s_3 = 6.8207 kJ/kg.K
$s_{4s} = s_3$; p_4 = 10 kPa i.e. 0.1 bar
$x_4 = 0.9$
$h_4 = h_f + x_4(h_v - h_f)$ = 191.81 + x_4(2583.9 - 191.81) = 2344.7 kJ/kg
h_{4s} = 2160.29 kJ/kg (using $x_{4s} = \frac{s_{4s} - s_f}{s_g - s_f} = 0.8229$)
$\eta_{is} = \frac{h_3 - h_4}{h_3 - h_{4s}} = 0.686$

(c) the power output of the steam turbine.
Solution:
$\dot{W}_{turb} = \dot{m}_g(h_3 - h_4)$ = 15.41 MW

(d) the thermal efficiency of the topping cycle (ratio of the steam turbine work output to the energy of the geothermal fluid relative to the standard ambient conditions, i.e. 25 °C and 1 atm.
Solution:
$\eta = \frac{\dot{m}_g(h_3 - h_4)}{\dot{m}_{ini}(h_1 - h_0)} = 0.076$; h_0 = 104.3 kJ/kg

(e) the mass flow rate of isobutane in the bottoming cycle.
Solution:
h_6 (from NIST) = 640.09 kJ/kg
h_6 (from NIST at 90 °C) =377.04 kJ/kg
$\dot{Q}_{in} = \dot{m}_l(h_6 - h_7)$ = 50.453 MW
h_8 (from NIST for isobutane) = 754.55 kJ/kg
h_9 (from NIST for isobutane) = 690.06 kJ/kg
h_{10} (from NIST for isobutane) = 270.2 kJ/kg
s_{10} (from NIST for isobutane) = 1.2424 kJ/kg.K
h_{11s} = 275.6 kJ/kg

$\eta_{pump} = \frac{h_{11s}-h_{10}}{h_{11}-h_{10}} = 0.90$ implies h_{11}=276.18 kJ/kg
and $\dot{Q}_{in} = 50.453MW = \dot{m}_{iso}(h_8 - h_{11})$ implies $\dot{m}_{iso} = 105.5$ kg/s

(f) the net power outputs of both the topping (flashing) and bottoming (binary) cycles of the plant.
Solution: Power output of Topping cycle = $\dot{W}_{turb}$ = 15.41 MW Power output of Bottoming cycle = $\dot{W}_{turbiso} = \dot{m}_{iso}(h_8 - h_9) = 6.8$ MW

(g) the thermal efficiencies of the bottoming (binary) cycle and of the combined plant.
Solution:
$\dot{W}_{pump} = \dot{m}_{iso}(h_{10} - h_{11})$ = -632.8 kW
$\eta = \frac{\dot{W}_{turbiso}}{\dot{Q}_{in}-\dot{W}_{pump}} = 0.1331$
$\eta_{tot} = \frac{\dot{W}_{turb}+\dot{W}_{turbiso}}{\dot{Q}_{ini}-\dot{W}_{pump}}$ =0.1087

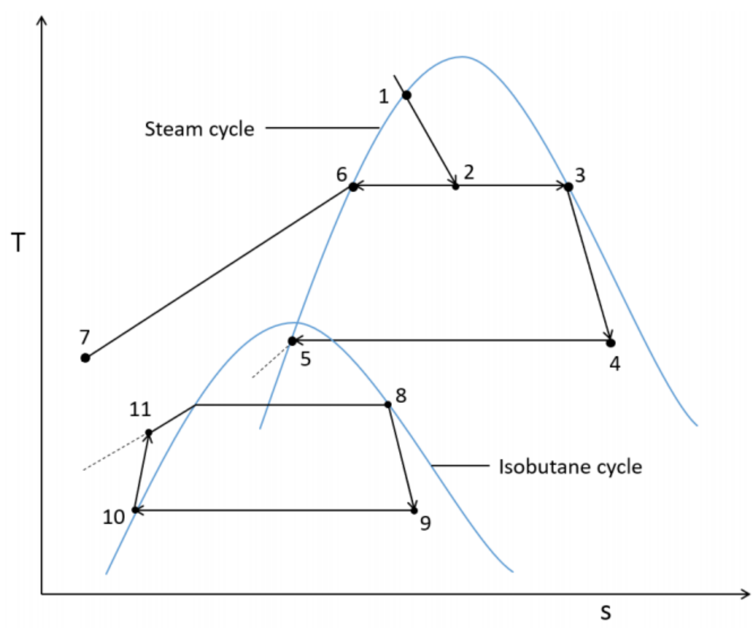

Figure 2: T-s diagram: Topping and bottoming cycle