Astrophysics IV : Stellar and galactic dynamics

Exercises

Problem 1 :

Demonstrate that the small radial motions around the guiding center in a weekly bared potential of pattern speed Ω_b is given by :

$$R_1\left(\theta_0(t)\right) = C_1 \cos\left(\frac{\chi_0\theta_0}{\Omega_0-\Omega_b}+\alpha\right) - \left[\frac{\partial\phi_b}{\partial R}+\frac{2\Omega_0\phi_b}{R\left(\Omega_0-\Omega_b\right)}\right]_{R_0}\frac{\cos\left(m\theta_0\right)}{\chi_0^2-m^2\left(\Omega_0-\Omega_b\right)^2}, \tag{1}$$

where :

$$\theta_0\left(t\right) = \left(\Omega_0-\Omega_b\right)t, \tag{2}$$

$$\kappa_0^2 = \left.\left(\frac{\partial^2\phi}{\partial R^2}+3\Omega^2\right)\right|_{R_0}, \tag{3}$$

is the radial epicycle frequency and Ω_0 is the circular frequency without the bar. Show that the small azimutal motion is given by :

$$\dot{\theta}_1(t) = -2\Omega_0\frac{R_1}{R_0} - \frac{\phi_b\left(R_0\right)}{R_0^2\left(\Omega_0-\Omega_b\right)}\cos\left(m\left(\Omega_0-\Omega_b\right)t\right) + const, \tag{4}$$

Hints : (1) Assume the rotating potential to be of the form :

$$\phi\left(R,\theta\right) = \phi_0\left(R\right)+\phi_1\left(R,\theta\right) = \phi_0(R)+\phi_b\left(R\right)\cos\left(m\theta\right), \tag{5}$$

with $\phi_1 \ll \phi_0$. (2) decompose the motion of the stars into two parts :

$$\begin{cases} R\left(t\right) = R_0 + R_1\left(t\right) \\ \theta\left(t\right) = \theta_0\left(t\right)+\theta_1\left(t\right) \end{cases} \tag{6}$$

with R_0 the radius of the guiding centre (circular orbit). (3) Develop the equations of motion to the first order.

Problem 2 :

Show from a simple geometrical argument that the density of the phase space is conserved for a 1-D harmonic oscillator.
Hint : demonstrate that particles move along ellipses in the phase space.