

Astrophysics IV: Stellar and galactic dynamics

Exercises**Problem 1:**

Show that the collisionless Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} + \mathbf{a} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0 \quad (1)$$

in

a) spherical coordinates is

$$\begin{aligned} \frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \frac{v_\theta}{r} \frac{\partial f}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial f}{\partial \phi} + \left[\frac{v_\theta^2 + v_\phi^2}{r} - \frac{\partial \Phi}{\partial r} \right] \frac{\partial f}{\partial v_r} + \\ + \left[\frac{1}{r} \left(\frac{v_\phi^2}{\tan \theta} - v_r v_\theta \right) \right] \frac{\partial f}{\partial v_\theta} - \frac{v_\phi}{r} \left(v_r + \frac{v_\theta}{\tan \theta} \right) \frac{\partial f}{\partial v_\phi} = 0 \end{aligned} \quad (2)$$

with $\Phi = \Phi(r)$.

b) in cylindrical coordinates is

$$\frac{\partial f}{\partial t} + v_r \frac{\partial f}{\partial r} + \frac{v_\phi}{r} \frac{\partial f}{\partial \phi} + v_z \frac{\partial f}{\partial z} + \left[\frac{v_\phi^2}{r} - \frac{\partial \Phi}{\partial r} \right] \frac{\partial f}{\partial v_r} - \frac{v_r v_\phi}{r} \frac{\partial f}{\partial v_\phi} - \frac{\partial \Phi}{\partial z} \frac{\partial f}{\partial v_z} = 0 \quad (3)$$

with $\Phi = \Phi(r, z)$.

Hints:

The gradients in spherical and cylindrical coordinates are given by

$$\begin{aligned} \nabla_{sph} f &= \frac{\partial f}{\partial r} \mathbf{e}_r + \frac{1}{r} \frac{\partial f}{\partial \theta} \mathbf{e}_\theta + \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi} \mathbf{e}_\phi \\ \nabla_{cyl} f &= \frac{\partial f}{\partial r} \mathbf{e}_r + \frac{1}{r} \frac{\partial f}{\partial \phi} \mathbf{e}_\phi + \frac{\partial f}{\partial z} \mathbf{e}_z \end{aligned}$$

To obtain expressions for the $\mathbf{a} = \dot{\mathbf{v}} = (\dot{v}_r, \dot{v}_\theta, \dot{v}_\phi)$ and $\dot{\mathbf{v}} = (\dot{v}_r, \dot{v}_\theta, \dot{v}_z)$, use the Euler-Lagrange equations of motion to find expressions for $v_r, v_\theta, v_\phi, v_z$ first.

Problem 2:

Derive the collisionless Boltzmann equation in cartesian coordinates in a frame that rotates with a constant angular velocity $\vec{\Omega}$. Write it in term of the effective potential.