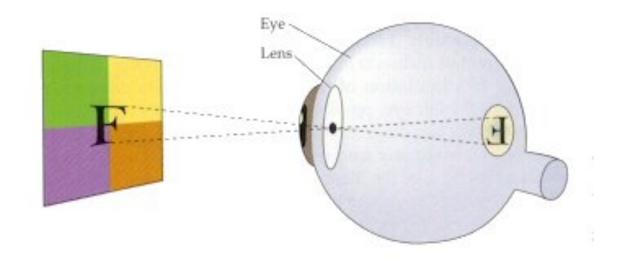
Image Formation (Cont'd) & Edge Detection

P. Fua (Taught by M. Salzmann) IC-CVLab EPFL



Reminder: Image Formation

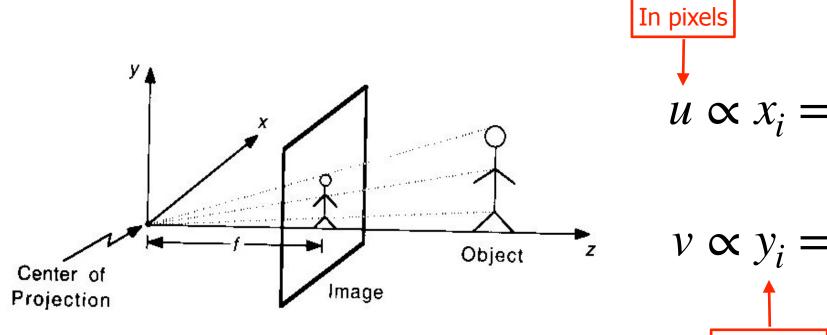


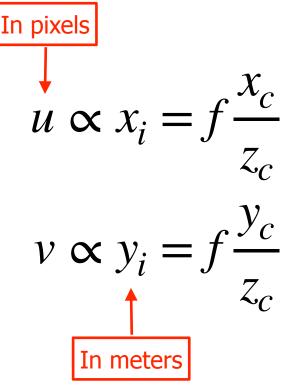
Projection from surfaces to 2-D sensor.

- Where: Geometry
- How bright: Radiometry
- Stored how: Sensing



Reminder: Pinhole Camera Model

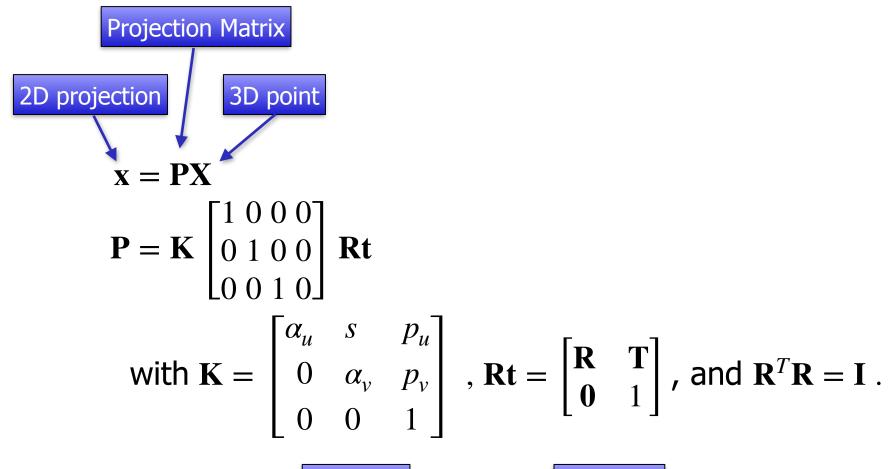




→ Reformulate it as a linear operation using homogeneous coordinates.



Reminder: Projection in Homogeneous Coordinates



Intrinsics

Extrinsics

Reminder: Camera Calibration

Internal Parameters:

- Horizontal and vertical scaling (2)
- Principal points (2)
- Skew of the axis (1)

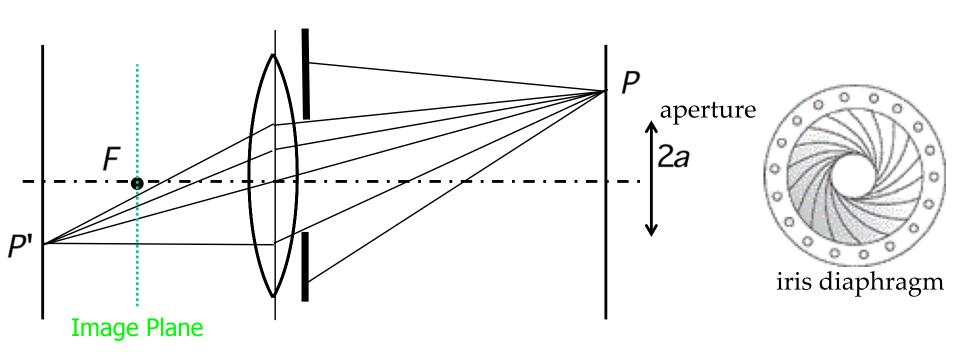
External Parameters:

- Rotations (3)
- Translations (3)

→There are 11 free parameters to estimate. This is known as calibrating the camera.



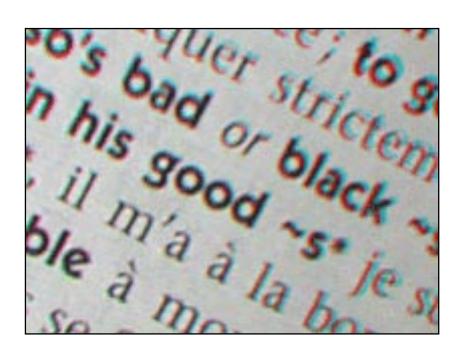
Reminder: Thin Lens



- Diameter d=2a of the lens that is exposed to light.
- The image plane is not located exactly where the rays meet.
- The greater a, the more blur there will be.



Reminder: Distortions



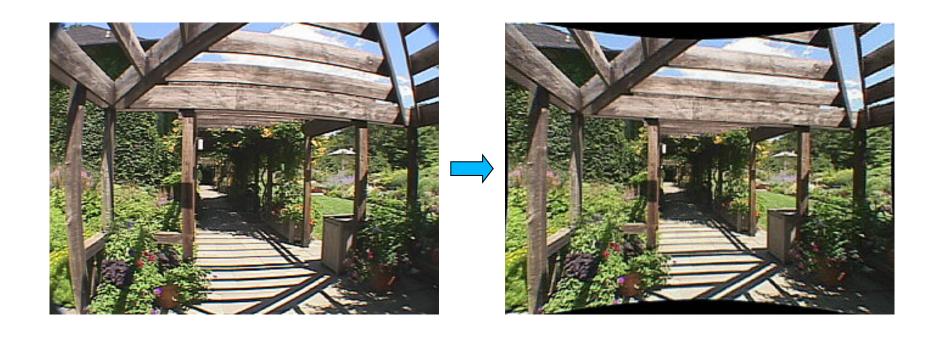


The lens is not exactly a "thin lens:"

- Different wave lengths are refracted differently,
- Barrel Distortion.



Undistorting

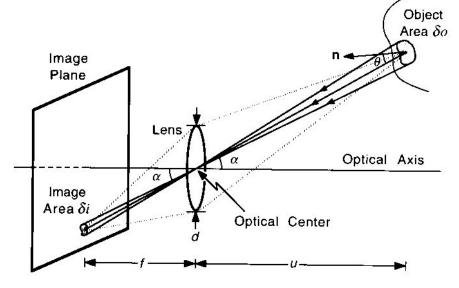


Once the image is undistorted, the camera projection can be formulated as a projective transform.

The pinhole camera model applies.



Fundamental Radiometric Equation



Scene Radiance (Rad): Amount of light radiation emitted from a surface point (Watt / m2 / Steradian).

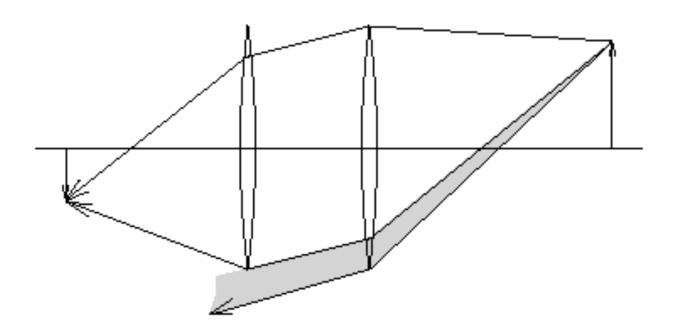
Image Irradiance (Irr): Amount of light incident at the image of the surface point (Watt / m2).

Irr =
$$\frac{\pi}{4} (\frac{d}{f})^2 \cos^4(\alpha) \text{Rad}$$
,

 \Rightarrow Irr \propto Rad for small values of α .



Vignetting



Images can get darker towards their edges because some of the light does not go through all the lenses.



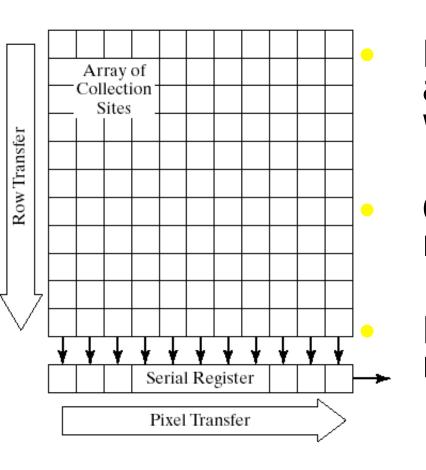
De Vignetting



—> As for geometric undistortion, undo vignetting to create an image that an ideal camera would have produced.



Sensor Array

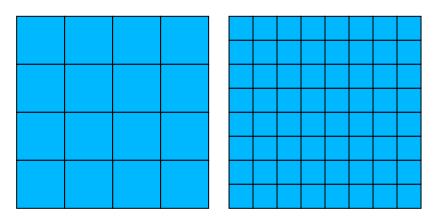


Photons free up electrons that are then captured by a potential well.

Charges are transferred row by row wise to a register.

Pixel values are read from the register.

Sensing



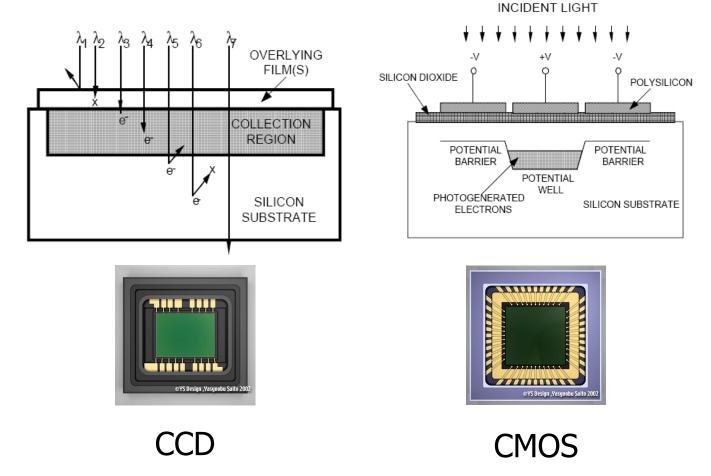
Conversion of the "optical image" into an "electrical image":

$$E(x,y) = \int_{t_0}^{t_1} \int_0^{\Lambda} Irr(x,y,t,\lambda) s(\lambda) dt d\lambda$$

$$I(m,n) = Quantize(\int_{x_0}^{x_1} \int_{y_0}^{y_1} E(x,y) dx dy)$$

- → Quantization in
- Time
- Space

Sensors



- Charged Coupling Devices (CCD): Made through a special manufacturing process that allows the conversion from light to signal to take place in the chip without distortion.
- Complimentary Metal Oxide Semiconductor (CMOS): Easier to produce and similar quality. Now used in most cameras except when quantum efficient pixels are needed, e.g. for astronomy.



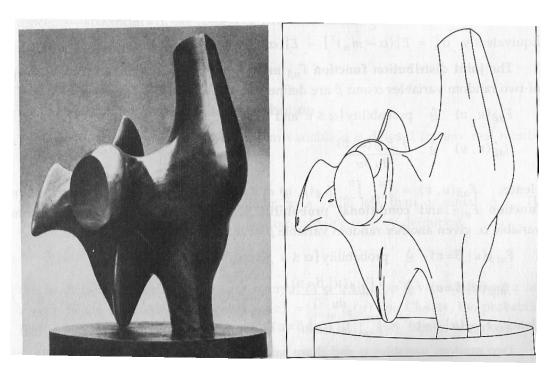
In Short

 Camera geometry can be modeled in terms of the pinhole camera model, which is linear in projective space.

 Image radiance is roughly proportional to surface radiance and the two can be used interchangeably for our purposes.



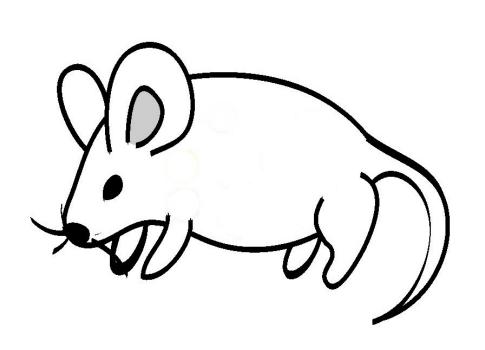
Edge Detection

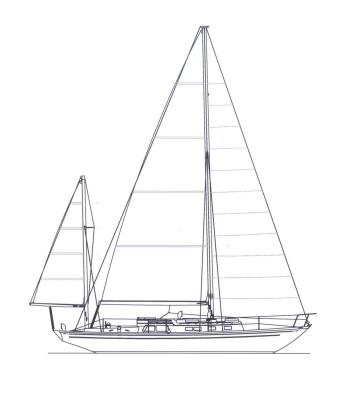


- What's an edge
- Image gradients
- Edge operators



Line Drawings

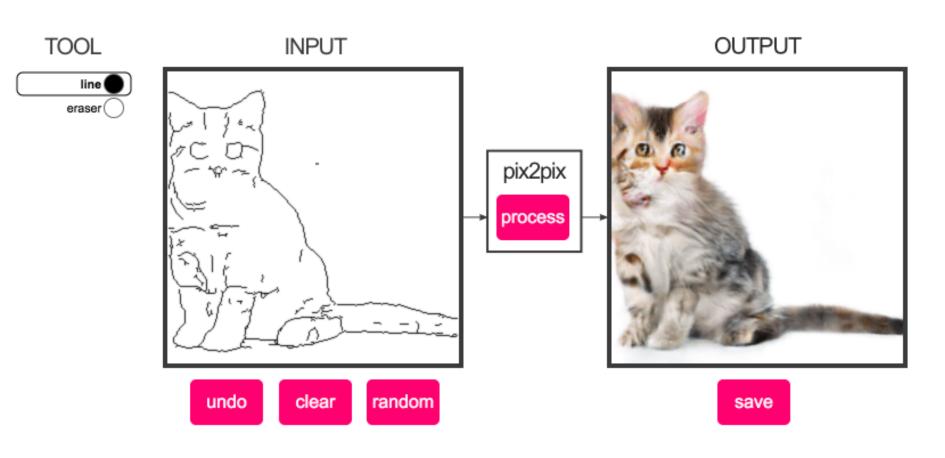




- Edges seem fundamental to human perception.
- They form a compressed version of the image.



From Edges To Cats



Deep-Learning based generative model.



Maps and Overlays





Corridor



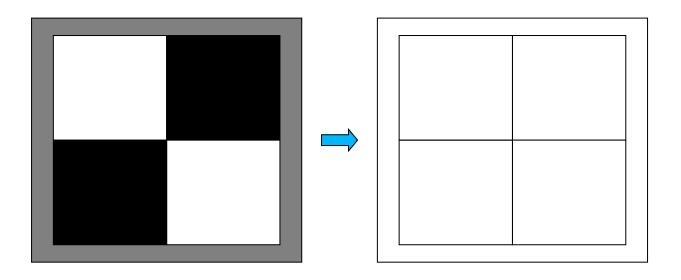


Corridor





Edges and Regions



Edges:

Boundary between bland image regions.

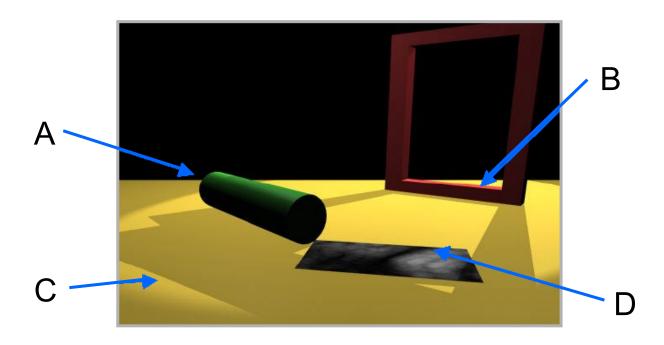
Regions:

Homogenous areas between edges.





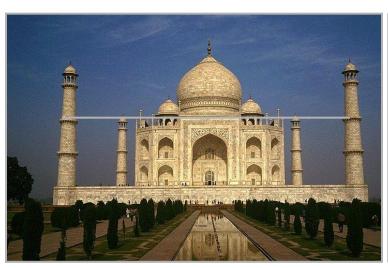
Discontinuities

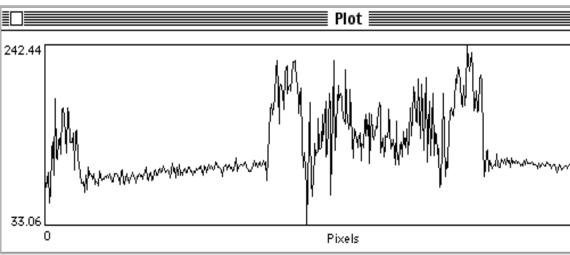


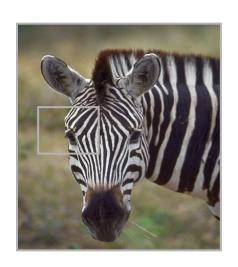
- A. Depth discontinuity: Abrupt depth change in the world
- B. Surface normal discontinuity: Change in surface orientation
- C. Illumination discontinuity: Shadows, lighting changes
- D. Reflectance discontinuity: Surface properties, markings
 - Sharply different Gray levels on both sides

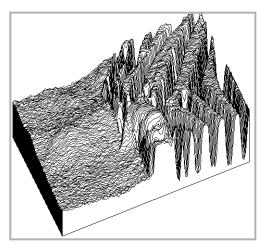


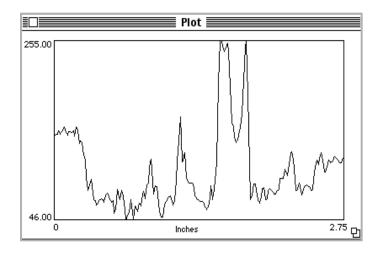
REALITY





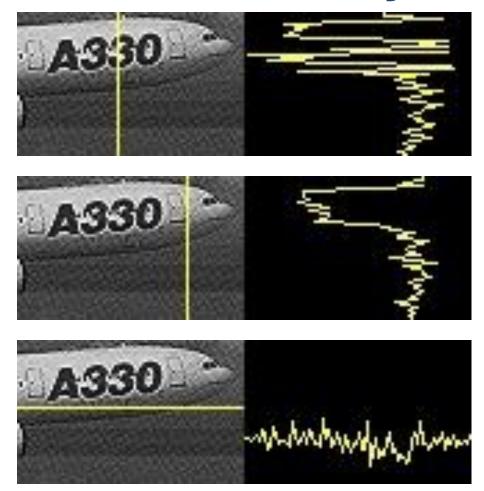








More Reality

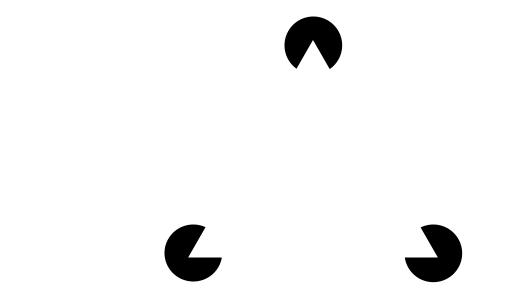


Very noisy signals

→ Prior knowledge is required!!



Optional: Illusory Contours



- No closed contour, but we still perceived an edge.
- This will not be further discussed in this class.



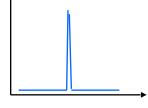
Ideal Step Edge

Rapid change in image => High local gradient

$$f(x) = step edge$$

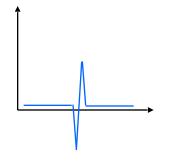
.

1st Derivative f'(x)



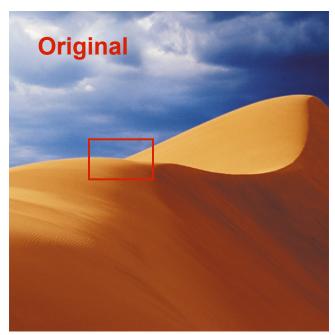
maximum

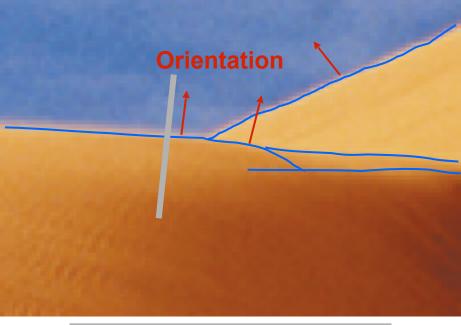
2nd Derivative f"(x)



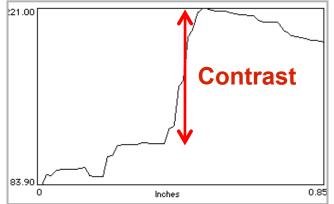
zero crossing

Edge Properties

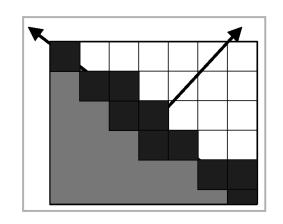








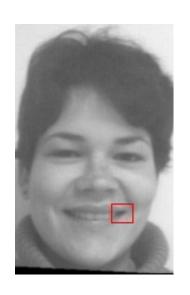
Edge Descriptors

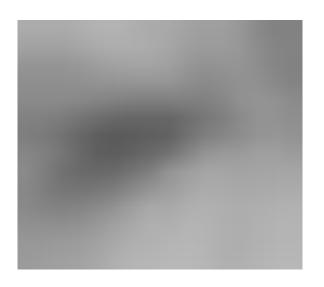


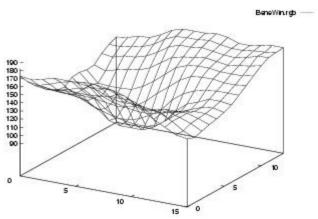
- Edge Normal:
 - Unit vector in the direction of maximum intensity change
- Edge Direction:
 - Unit vector perpendicular to the edge normal
- Edge position or center
 - Image location at which edge is located
- Edge Strength
 - Speed of intensity variation across the edge.



Images as 3-D Surfaces

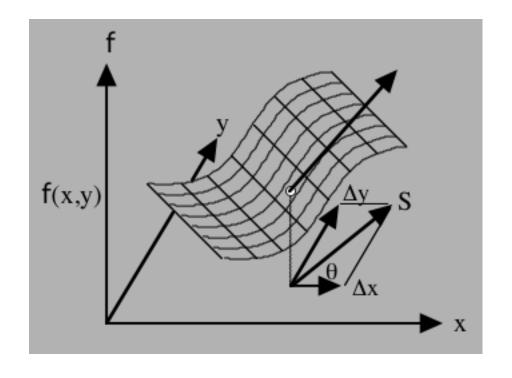








Geometric Interpretation



Since I(x,y) is not a continuous function:

- 1.Locally fit a smooth surface.
- 2. Compute its derivatives.

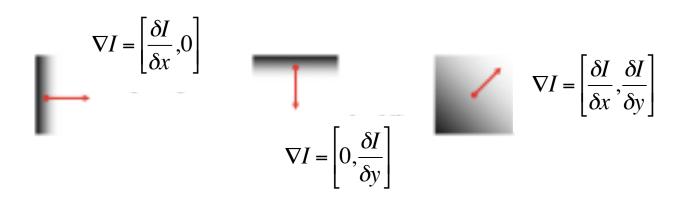


Image Gradient

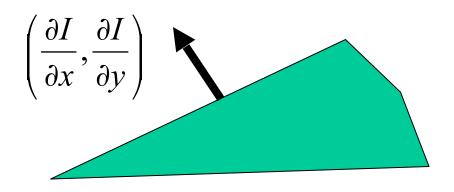
The gradient of an image

$$\nabla I = \left[\frac{\delta I}{\delta x}, \frac{\delta I}{\delta y} \right]$$

points in the direction of most rapid change in intensity.



Magnitude And Orientation

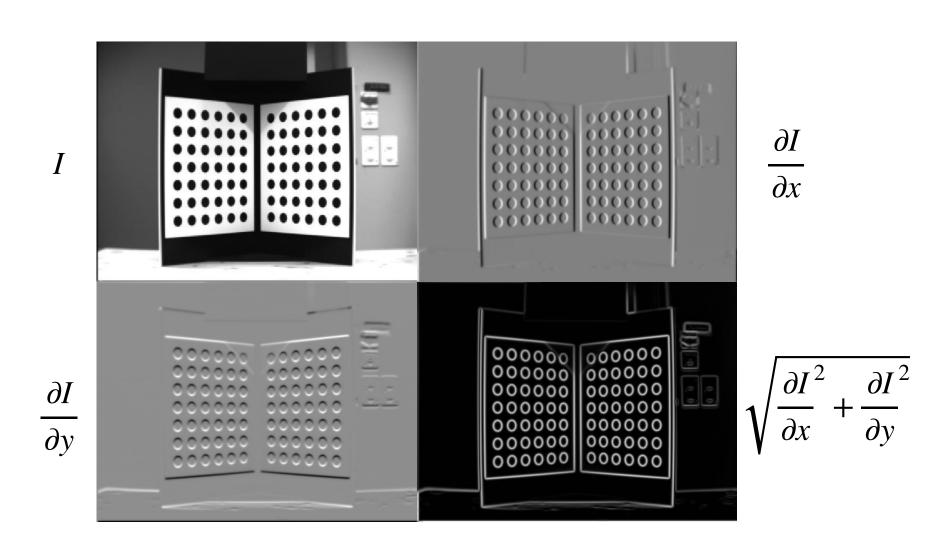


Measure of contrast :
$$G = \sqrt{\frac{\partial I}{\partial x}^2 + \frac{\partial I}{\partial y}^2}$$

Edge orientation : $\theta = \arctan(\frac{\partial I}{\partial y}, \frac{\partial I}{\partial x})$



Gradient Images



The gradient magnitude is unaffected by orientation



Real Images



... but not directly usable in most real-world images.

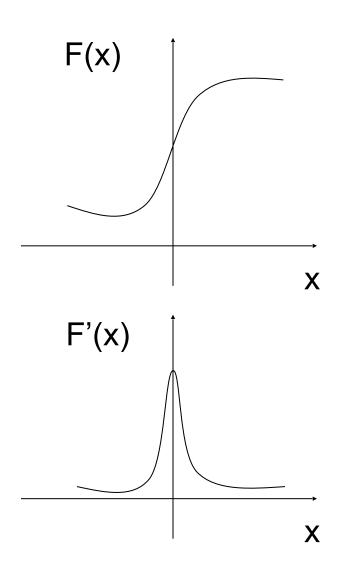


Edge Operators

- Difference Operators
- Convolution Operators
- Trained Detectors
- Deep Nets



Gradient Methods



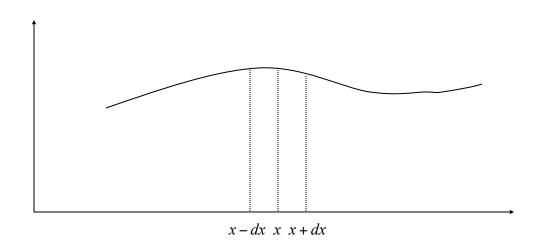
Edge = Sharp variation



Large first derivative

1D Finite Differences

In one dimension:



$$\frac{df}{dx} \approx \frac{f(x+dx) - f(x)}{dx} \approx \frac{f(x+dx) - f(x-dx)}{2dx}$$

$$\frac{d^2f}{dx^2} \approx \frac{f(x+dx) - 2f(x) + f(x-dx)}{dx^2}$$



Coding 1D Finite Differences

Line stored as an array:

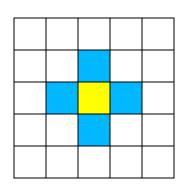


for i in range(n-1):q[i]=(p[i+1]-p[i])

for i in range(1,n-1):q[i]=(p[i+1]-p[i-1])/2

• q=(p[2:]-p[:-2])/2

2D Finite Differences

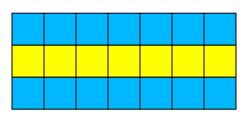


$$\frac{\partial f}{\partial x} \approx \frac{f(x+dx,y) - f(x,y)}{dx} \approx \frac{f(x+dx,y) - f(x-dx,y)}{2dx}$$

$$\frac{\partial f}{\partial y} \approx \frac{f(x,y+dy) - f(x,y)}{dy} \approx \frac{f(x,y+dy) - f(x,y-dy)}{2dy}$$



Coding 2D Finite Differences





Python

C

Image stored as a 2D array:

•
$$dx = p[1:,:]-p[:-1,:]$$

 $dy = p[:,1:]-p[:,:-1]$

```
• dx = (p[2:,:]-p[:-2,:])/2

dy = (p[:,2:]-p[:,:-2])/2
```

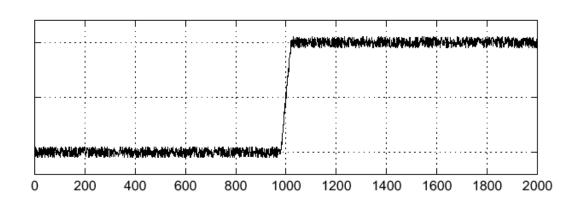
```
Image stored in raster format:
{
   int i;
   for(i=0;i<xdim;i++){
      dx[i] = p[i+1] -p[i];
      dy[i] = p[i+xdim]-p[i];
   }
}</pre>
```

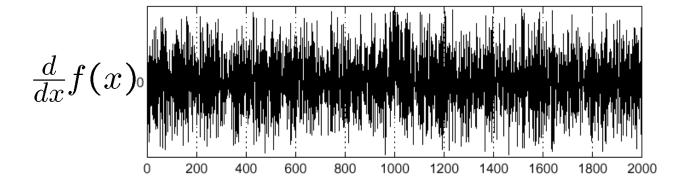
- Only 1D array accesses
- No multiplications
- -> Can be exploited to increase speed.



Noise in 1D

Consider a single row or column of the image:







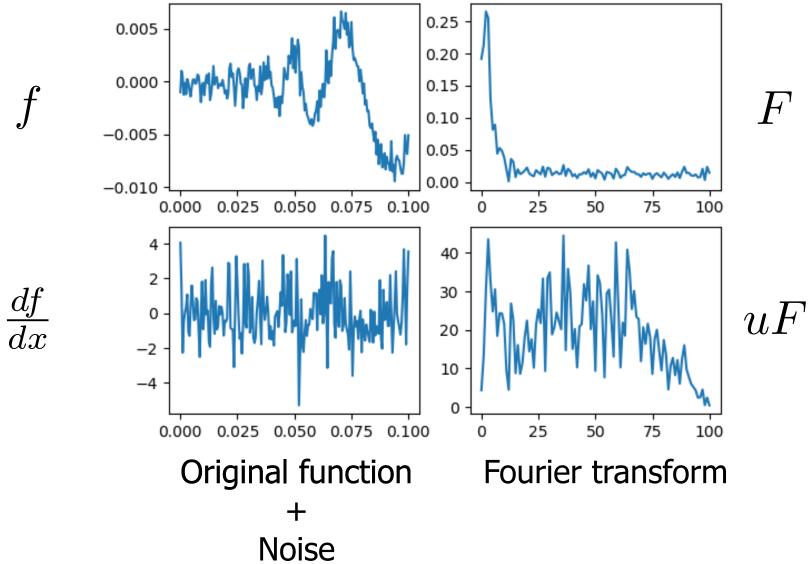
Fourier Interpretation

Function	Fourier Transform
$\frac{df}{dx}(x)$	uF(u)
$\frac{\delta f}{\delta x}(x,y)$	uF(u,v)
$\frac{\delta f}{\delta y}(x,y)$	vF(u,v)

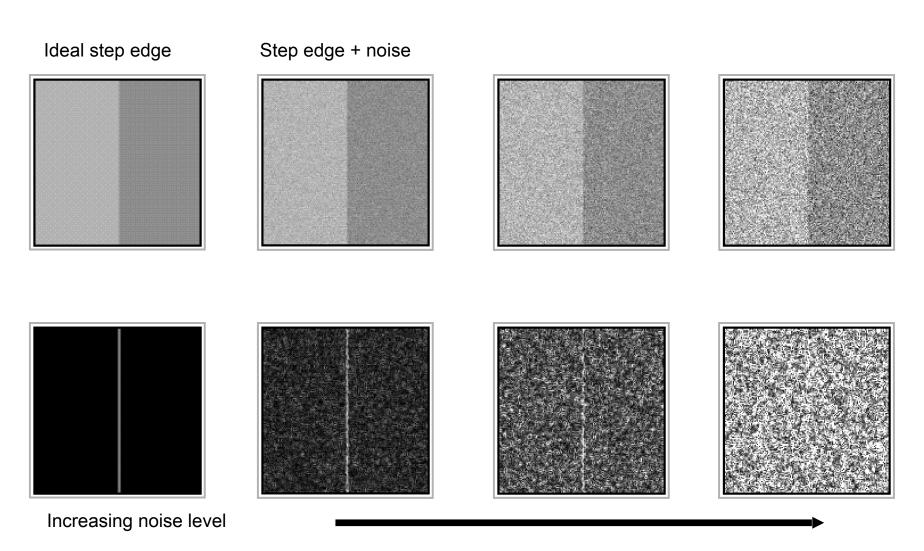
Differentiating emphasizes high frequencies and therefore noise!



$f(x) = x^2 \sin(1/x)$



Noise in 2D



As the amount of noise increases, the derivatives stop being meaningful.

EPF

Removing Noise

Problem:

High frequencies and differentiation do not mix well.

Solution:

- Suppress high frequencies by
 - using the Discrete Fourier Transform.



Discrete Fourier Transform

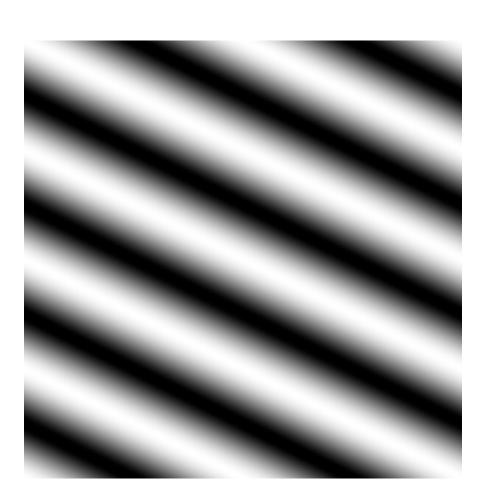
$$F(\mu, \nu) = \frac{1}{\sqrt{M*N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-2i\pi(\mu x/M + \nu y/N)}$$
$$f(x, y) = \frac{1}{\sqrt{M*N}} \sum_{\mu=0}^{M-1} \sum_{\nu=0}^{N-1} F(\mu, \nu) e^{+2i\pi(\mu x/M + \nu y/N)}$$

The DFT is the discrete equivalent of the 2D Fourier transform:

- The 2D function f is written as a sum of sinusoids.
- The DFT of f convolved with g is the product of their DFTs.



Fourier Basis Element



Real part of

$$e^{+2i\pi(ux+vy)}$$

where

- $\sqrt{u^2 + v^2}$ represents the frequency,
- atan(v, u) represents the orientation.

Fourier Basis Element



Real part of

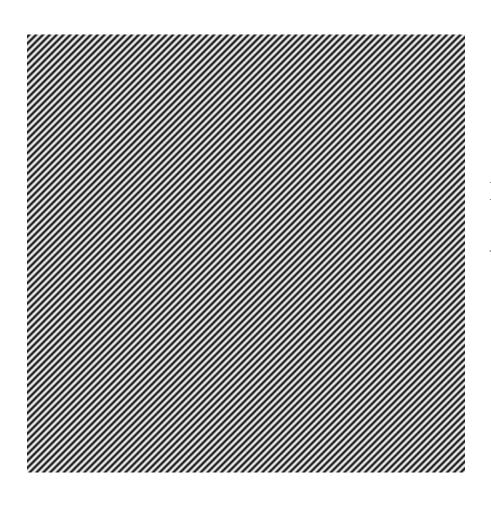
$$e^{+2i\pi(ux+vy)}$$

where

• $\sqrt{u^2 + v^2}$ is larger than before.



Fourier Basis Element



Real part of

 $e^{+2i\pi(ux+vy)}$

where

• $\sqrt{u^2 + v^2}$ is larger still.

Truncated Inverse DFT

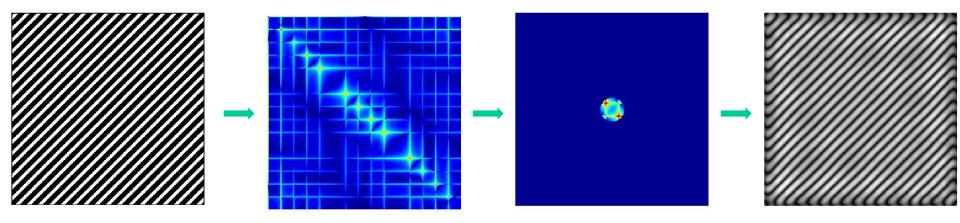
$$F(\mu,\nu) = \frac{1}{\sqrt{M*N}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-2i\pi(\mu x/M + \nu y/N)}$$
$$f(x,y) = \frac{1}{\sqrt{M*N}} \sum_{\mu=0}^{M-1} \sum_{\nu=0}^{N-1} F(\mu,\nu) e^{+2i\pi(\mu x/M + \nu y/N)}$$

$$f(x,y) = \frac{1}{\sqrt{M*N}} \sum_{\mu^2 + \nu^2 < T} F(\mu,\nu) e^{+2i\pi(\mu x/M + \nu y/N)}$$
T is a hand-specified threshold.

- The sinusoids corresponding to $\mu^2 + \nu^2 \ge T$ depict high frequencies.
- Removing them amounts to removing high-frequencies.



Smoothing by Truncating the IDFT



Rotated stripes:

- Dominant diagonal structures
- Discretization produces additional harmonics
- —> Removing higher frequencies and reconstructing yields a smoothed image.



Removing Noise

Problem:

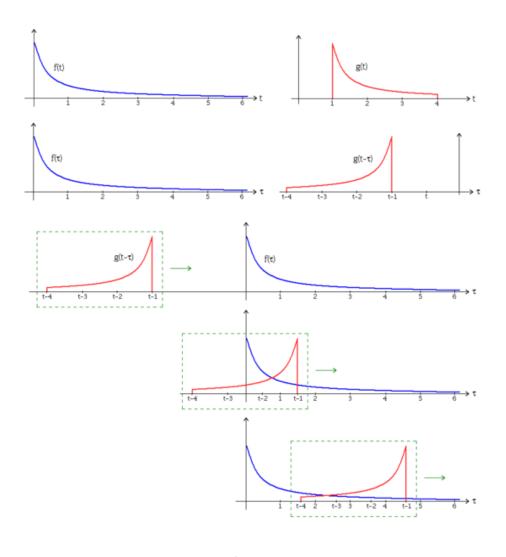
High frequencies and differentiation do not mix well.

Solution:

- Suppress high frequencies by
 - using the Discrete Fourier Transform,
 - convolving with a low-pass filter.



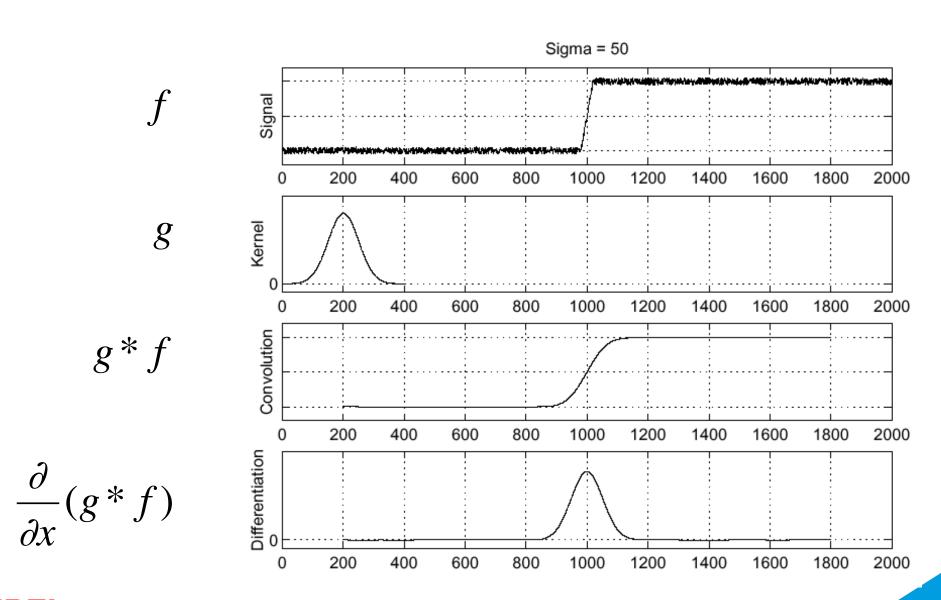
1D Convolution



$$g * f(t) = \int_{\tau} g(t - \tau) f(\tau) d\tau$$

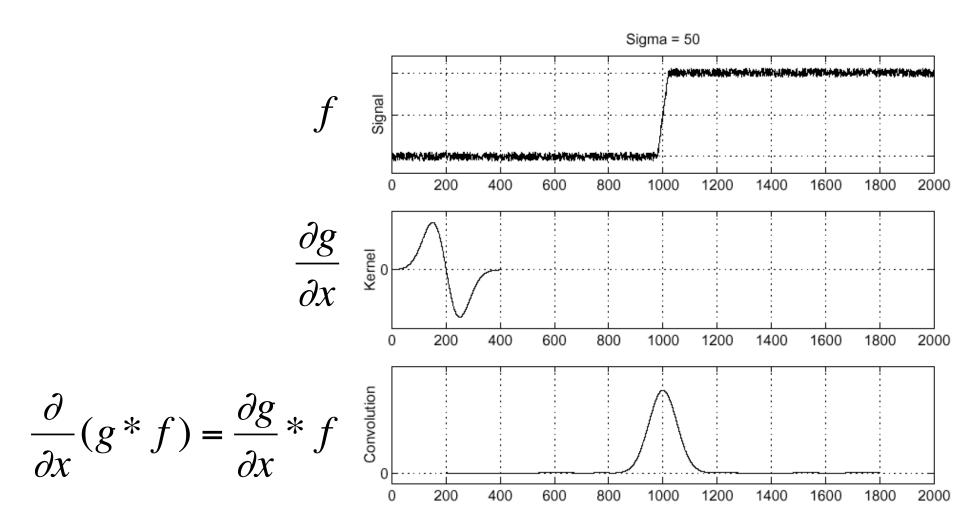


Smooth Before Differentiating



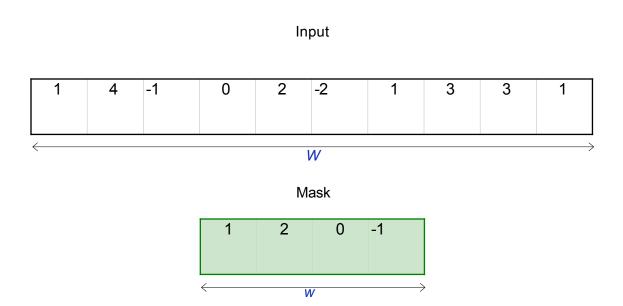


Simultaneously Smooth and Differentiate

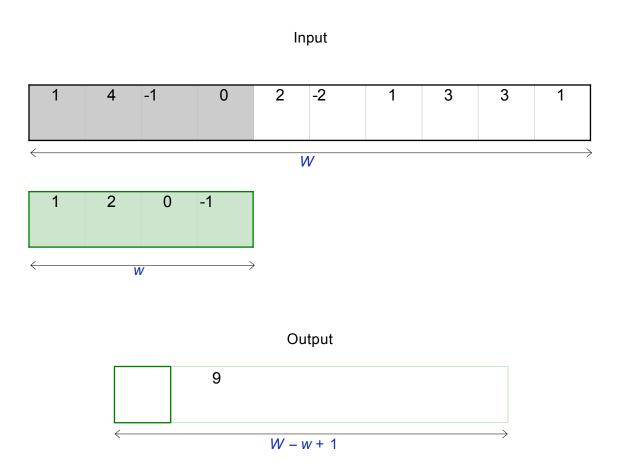


--> Faster because dg/dx can be precomputed.

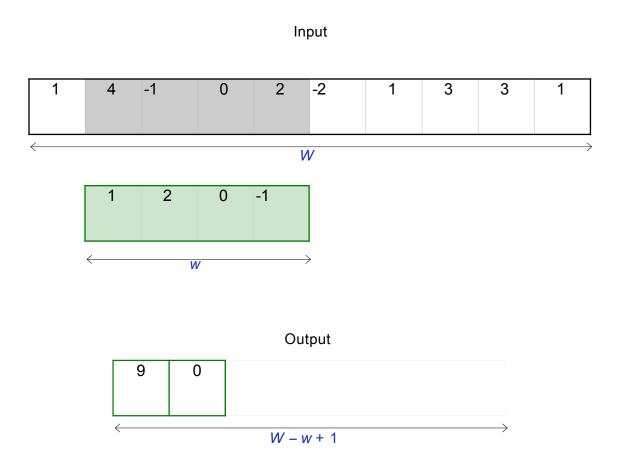




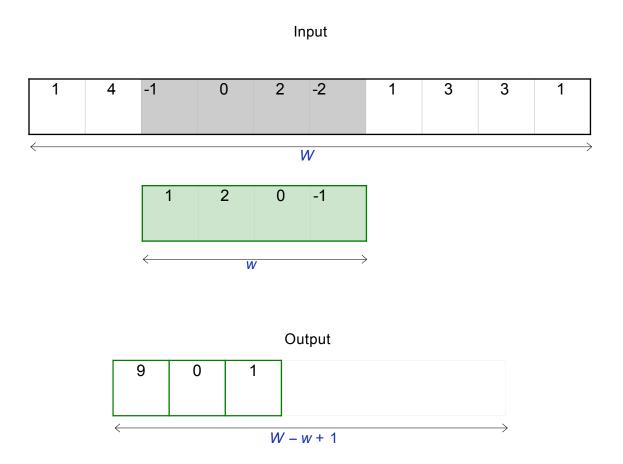




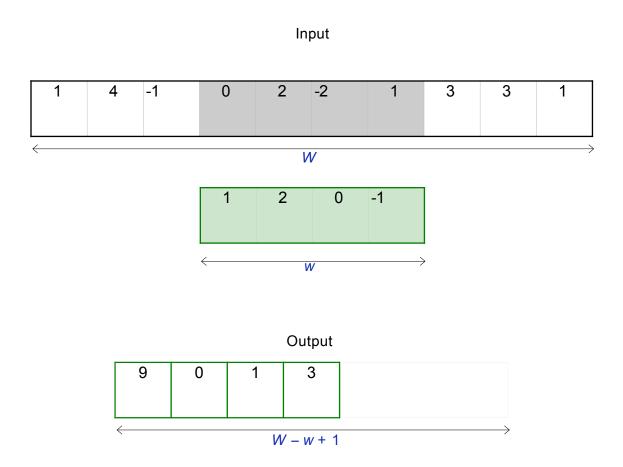




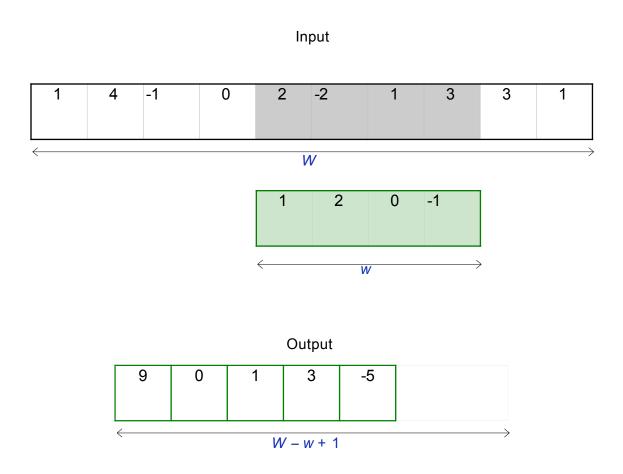




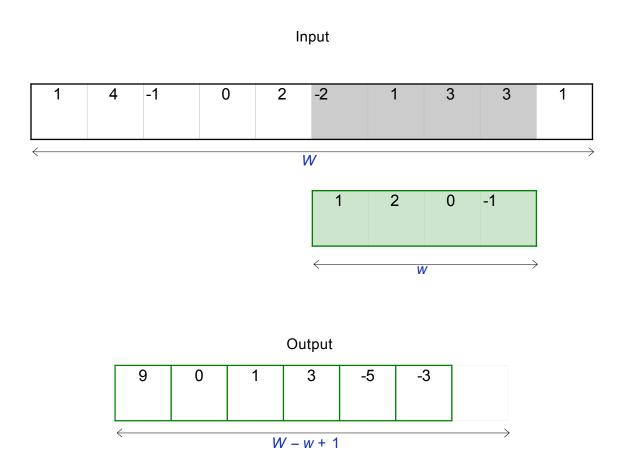




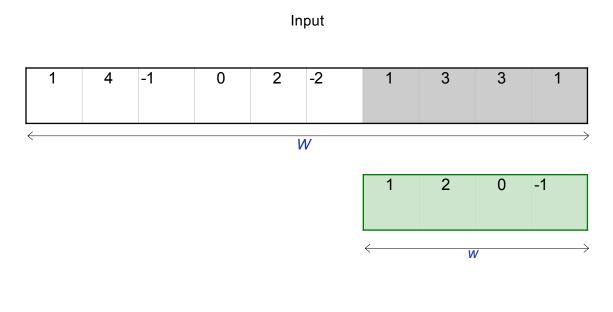


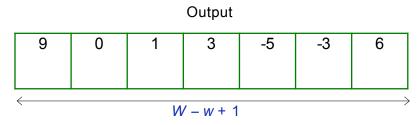












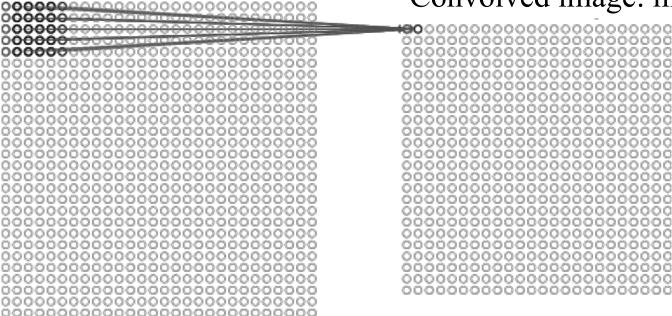


Input 0 2 -2 3 -1 3 4 1 Mask 0 -1 \mathbf{m} Output 0 -3 6 m*f W - w + 1 $m * f(x) = \sum_{i=0}^{w} m(i)f(x-i)$



Input image: f

Convolved image: m**f



Convolution mask m, also known as a kernel.

$$\begin{bmatrix} m_{11} & \dots & m_{1w} \\ \dots & \dots & \dots \\ m_{w1} & \dots & m_{ww} \end{bmatrix}$$

$$m * *f(x, y) = \sum_{i=0}^{w} \sum_{j=0}^{w} m(i, j) f(x - i, y - j)$$

Differentiation As Convolution

$$\begin{bmatrix} -1,1 \end{bmatrix} \rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x+dx,y) - f(x,y)}{dx}$$

$$\begin{bmatrix} -0.5,0,0.5 \end{bmatrix} \rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x+dx,y) - f(x-dx,y)}{2dx}$$

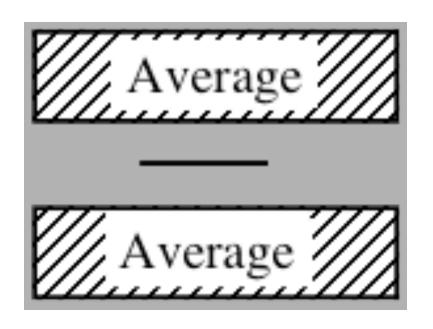
$$\begin{bmatrix} -1 \\ 1 \end{bmatrix} \rightarrow \frac{\partial f}{\partial y} \approx \frac{f(x,y+dy) - f(x,y)}{dy}$$

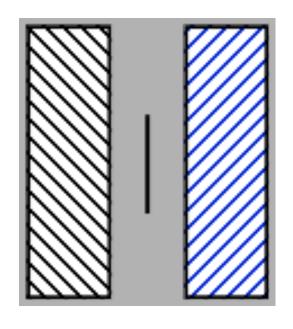
$$\begin{bmatrix} -0.5 \\ 0 \\ 0.5 \end{bmatrix} \rightarrow \frac{\partial f}{\partial y} \approx \frac{f(x,y+dy) - f(x,y-dy)}{2dy}$$

→ Use wider masks to add some smoothing



Smoothing and Differentiating

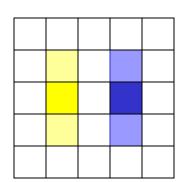




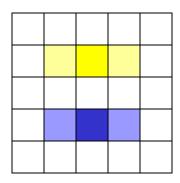
Compute the difference of averages on either side of the central pixel.



3X3 Masks



x derivative y derivative



$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

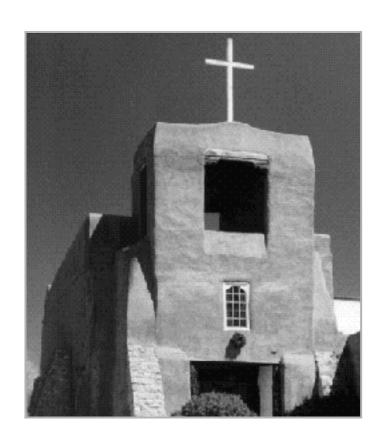
$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Prewitt operator

Sobel operator



Prewitt Example



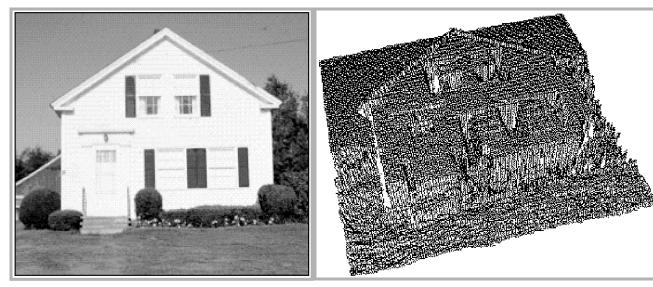
Santa Fe Mission



Gradient Image



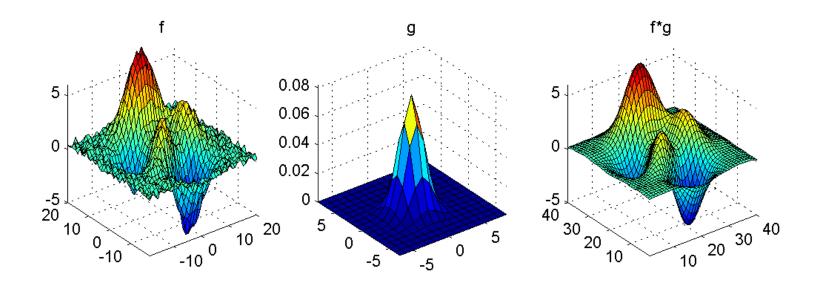
Sobel Example







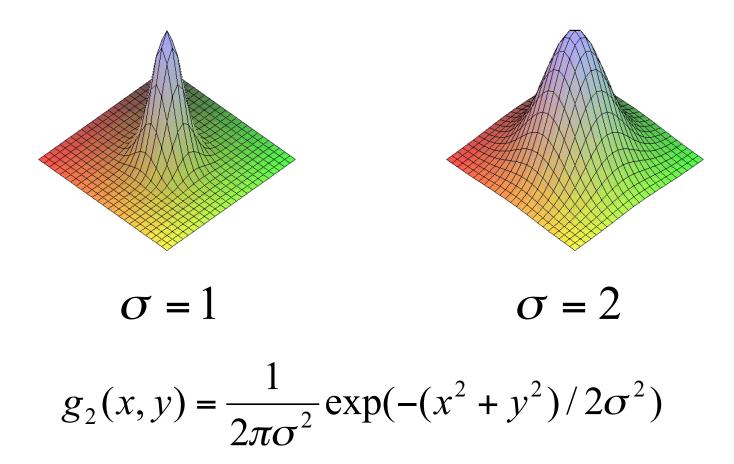
Gaussian Smoothing



- More principled way to eliminate high frequency noise.
- Is fast because the kernel is
 - small,
 - separable.



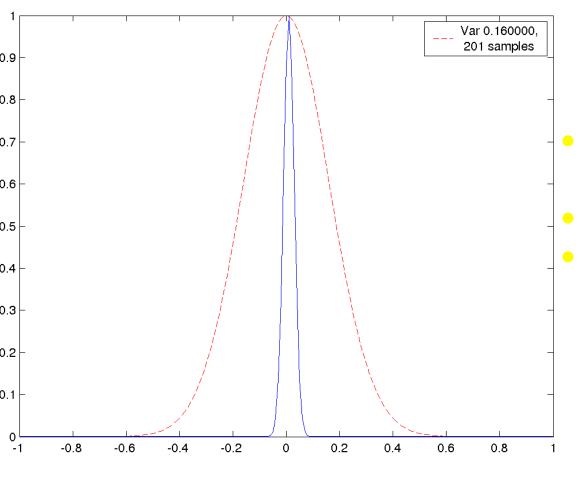
Gaussian Functions



- The integral is always 1.0
- The larger σ , the broader the Gaussian is.
- As σ approaches 0, the Gaussian approximates a Dirac function.



DFT of a Gaussian



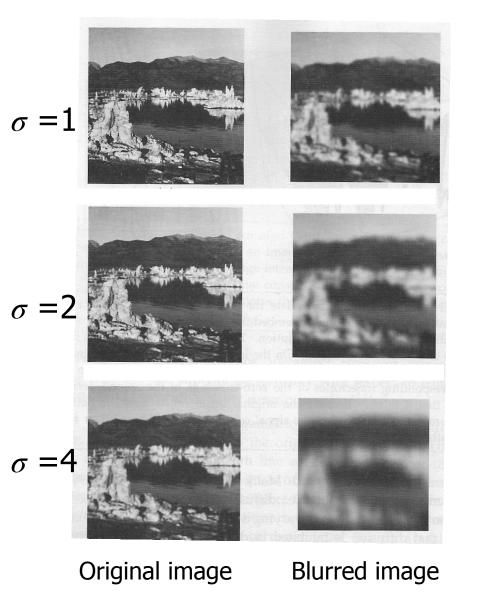
- The DFT of a Gaussian is a Gaussian.
- It has finite support.
- Its width is inversely proportional to that of the original Gaussian.

Gaussians as Low-Pass Filters

- The Fourier transform of a convolution is the product of their Fourier transforms: $\mathcal{F}(g * f) = \mathcal{F}(g)\mathcal{F}(f)$.
- If g is a Gaussian, so is $\mathcal{F}(g)$.
- Furthermore if g is broad, the support of $\mathcal{F}(g)$ is small.
- So is the support of $\mathcal{F}(g * f)$.
- There are no more high-frequencies in g * f.

—> Convolving with a Gaussian suppresses the high frequencies.

Gaussian Smoothed Images



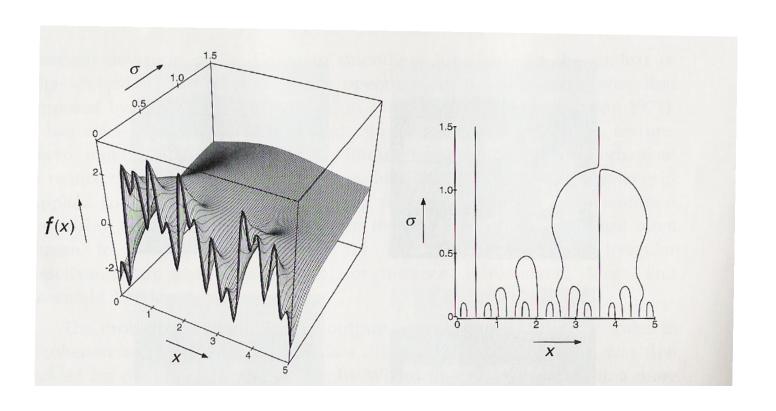


Blur σ

Blur $\sigma = 4$

Blur $\sigma = 2$

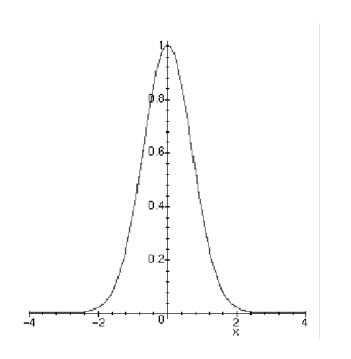
Scale Space



Increasing scale (σ) removes high frequencies (details) but never adds artifacts.



Separability



$$g_1(x) = \frac{1}{\sqrt{\pi}\sigma} \exp(-x^2/\sigma^2)$$
$$g_2(x, y) = g_1(x)g_1(y)$$

$$\int_{u} \int_{v} g_{2}(u, v) f(x - u, y - v) du dv = \int_{u} g_{1}(u) (\int_{v} g_{1}(v) f(x - u, y - v) dv) du$$

$$= \int_{v} g_{1}(v) (\int_{u} g_{1}(u) f(x - u, y - v) du) dv$$

-> 2D convolutions are never required. Smoothing can be achieved by successive 1D convolutions, which is faster.



Continuous Gaussian Derivatives

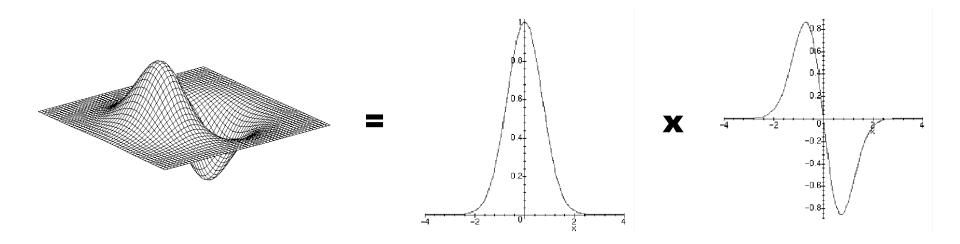


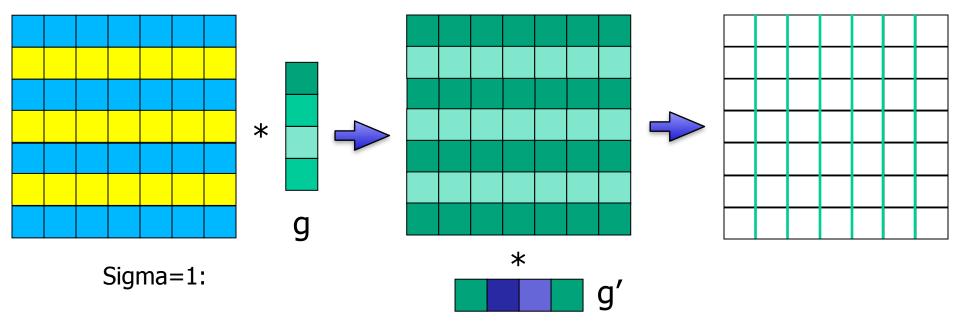
Image derivatives computed by convolving with the derivative of a Gaussian:

$$\frac{\partial}{\partial x} \int_{u} \int_{v} g_{2}(u,v) f(x-u,y-v) du dv = \int_{u} g_{1}(u) \left(\int_{v} g_{1}(v) f(x-u,y-v) dv \right) du$$

$$\frac{\partial}{\partial v} \int \int g_2(u,v) f(x-u,y-v) du dv = \int_v g_1'(v) \left(\int_u g_1(u) f(x-u,y-v) du \right) dv$$



Discrete Gaussian Derivatives



g: 0.000070 0.010332 0.207532 0.564131 0.207532 0.010332 0.000070

g': 0.000418 0.041330 0.415065 0.000000 -0.415065 -0.041330 -0.000418

Sigma=2:

g: 0.005167 0.029735 0.103784 0.219712 0.282115 0.219712 0.103784 0.029735 0.005167

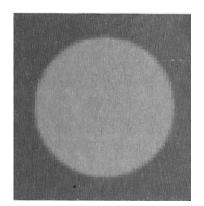
g': 0.010334 0.044602 0.103784 0.109856 0.000000 -0.109856 -0.103784 -0.044602 -0.010334

—> Only requires 1D convolutions with relatively small masks.

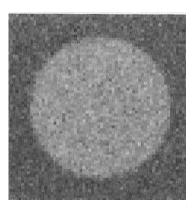


Increasing Sigma

Input Images

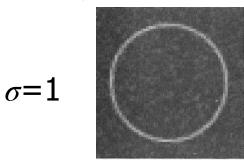


No Noise

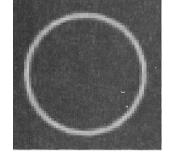


Noise Added

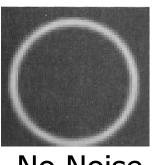
Gradient Images



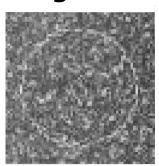




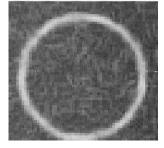
 σ =4



No Noise





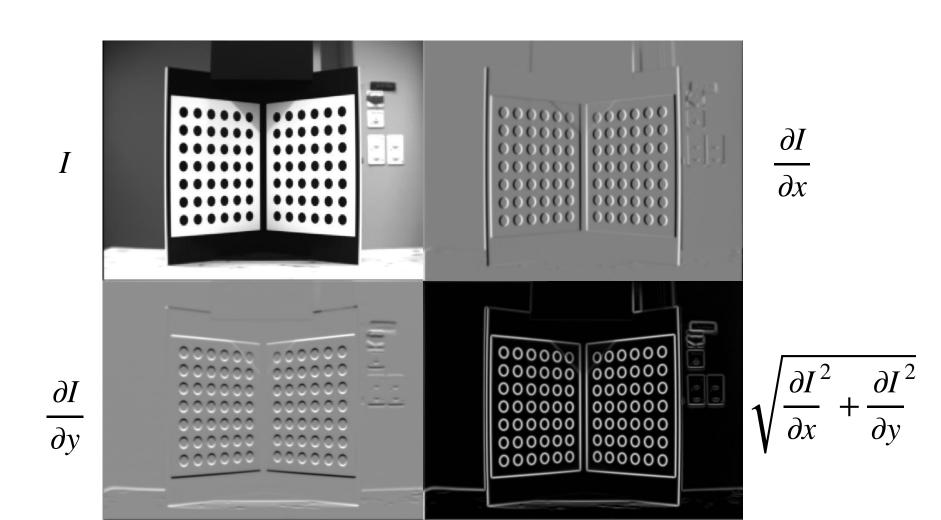


Noise Added

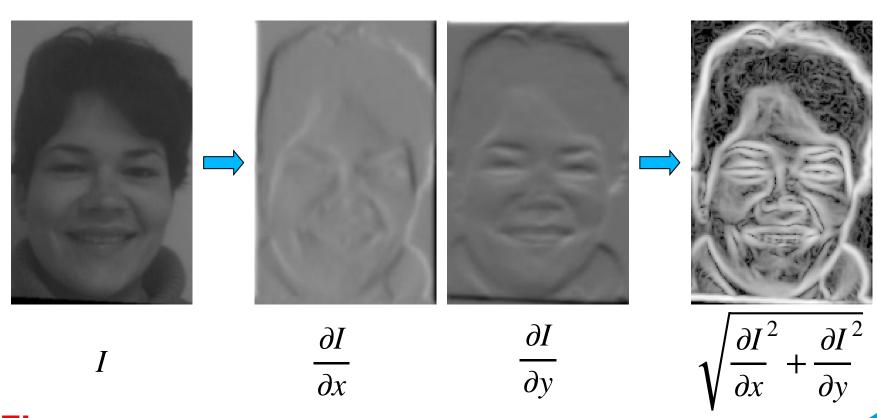
—> Larger sigma values improve robustness but degrade precision.



Derivative Images

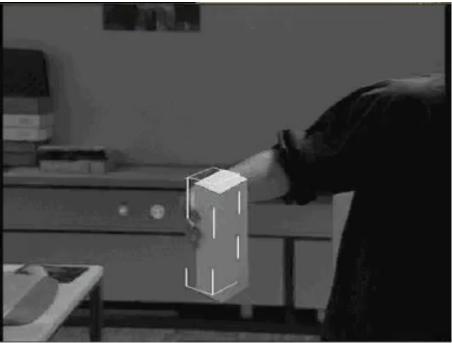


Derivative Images



Gradient-Based Tracking

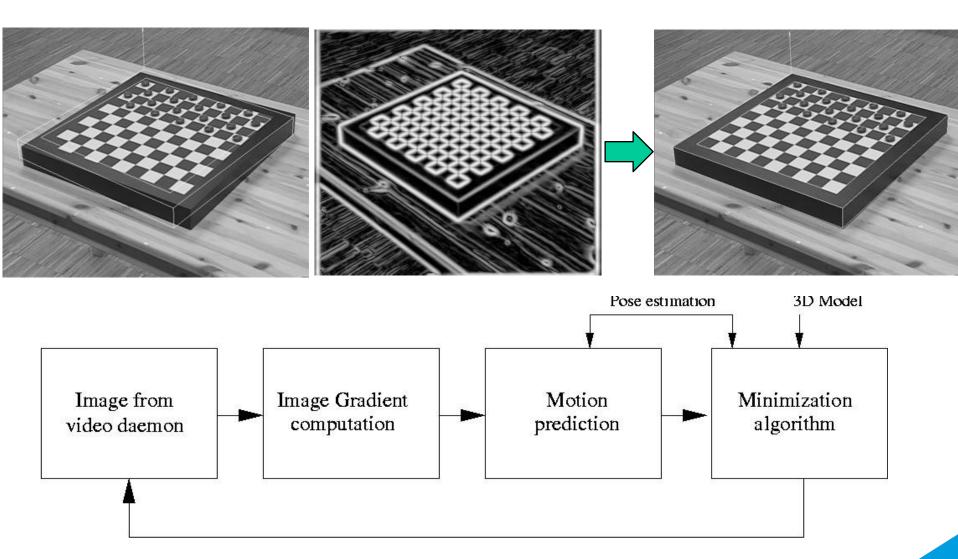




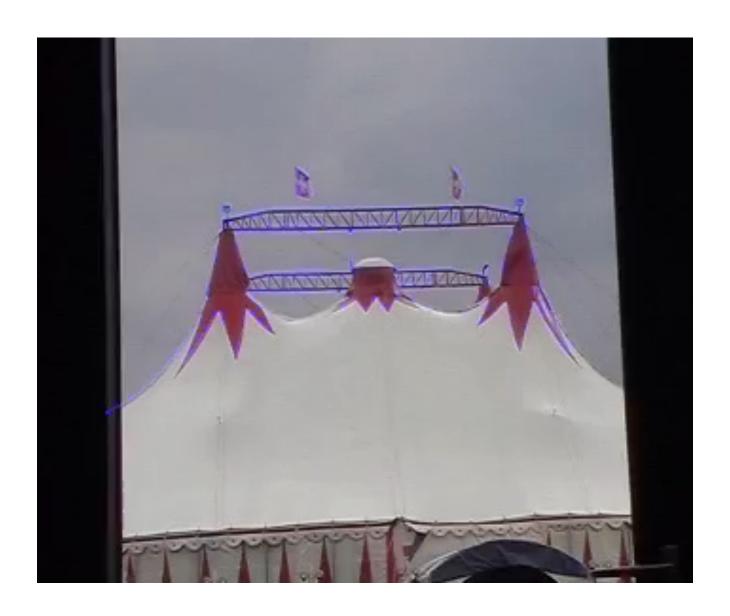
Maximize edge-strength along projection of the 3—D wireframe.



Gradient Maximization



Real-Time Tracking



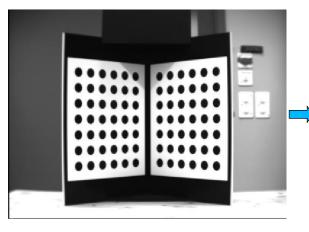


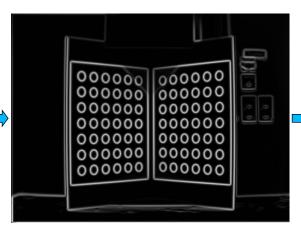
Canny Edge Detector

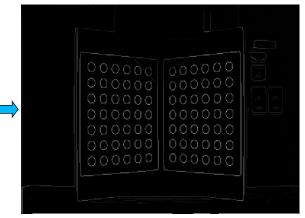
1

$$\sqrt{\frac{\partial I^2}{\partial x} + \frac{\partial I^2}{\partial y}}$$

Thinned gradient image















Canny Edge Detector



Convolution

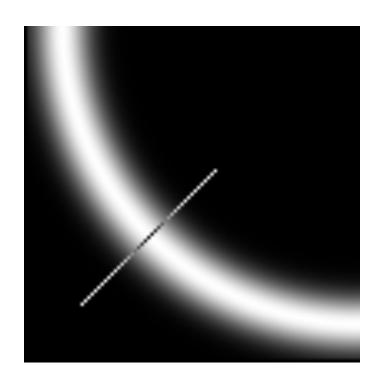
- Gradient strength
- Gradient direction

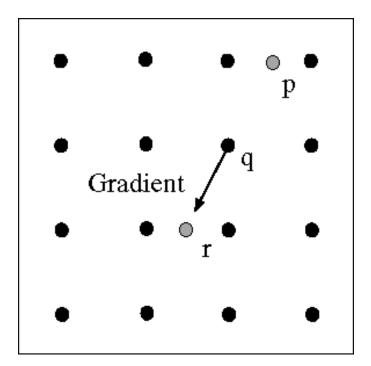
Thresholding

Non Maxima Suppression Hysteresis Thresholding



Non-Maxima Suppression

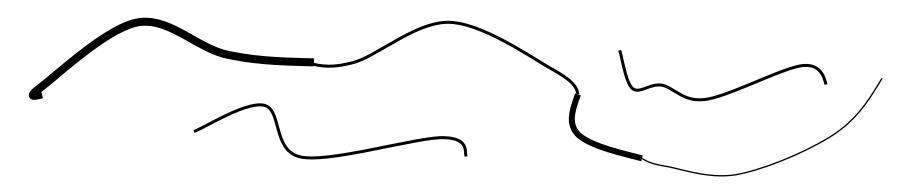




Check if pixel is local maximum along gradient direction, which requires checking interpolated pixels p and r.



Hysteresis Thresholding



- Algorithm takes two thresholds: high & low
 - A pixel with edge strength above high threshold is an edge.
 - Any pixel with edge strength below low threshold is not.
 - Any pixel above the low threshold and next to an edge is an edge.
- Iteratively label edges
 - Edges grow out from 'strong edges'
 - Iterate until no change in image.



Canny Results





 σ =1, T2=255, T1=1

'Y' or 'T' junction problem with Canny operator



Canny Results



σ=1, T2=255, T1=220

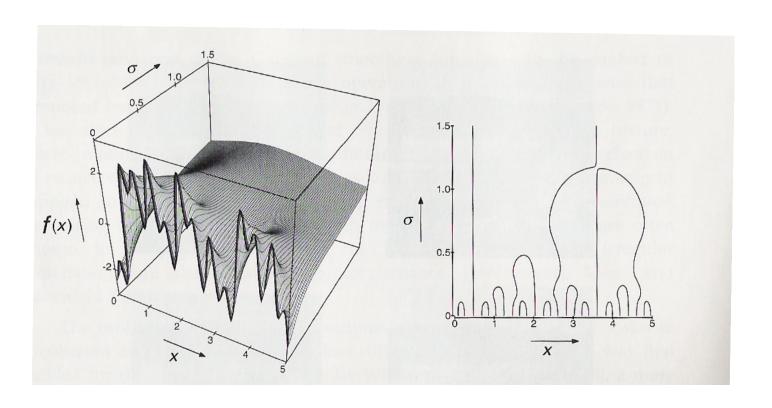


σ=1, T2=128, T1=1



 σ =2, T2=128, T1=1

Scale Space Revisited



Increasing scale (o) removes details but never adds new ones:

- Edge position may shift.
- Two edges may merge.
- An edge may **not** split into two.



Multiple Scales







→ Choosing the right scale is a difficult semantic problem.

Scale vs Threshold



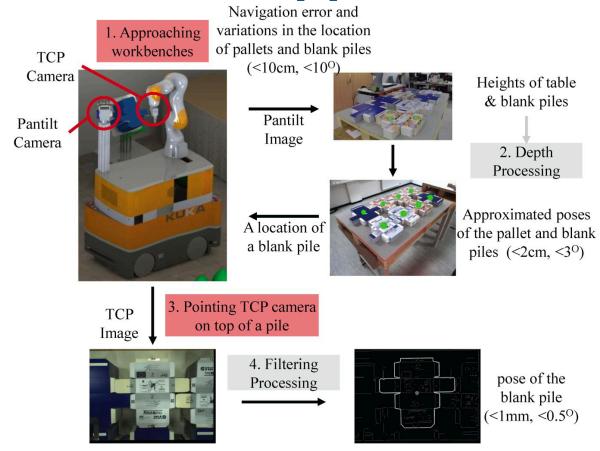
Fine scale High threshold

Coarse scale High threshold

Coarse scale Low threshold



Industrial Application



In industrial environments where the Canny parameters can be properly adjusted:

- It is fast.
- Does not require training data.



Visual Servoing



—> A useful tool in our toolbox.



Tracking a Rocket



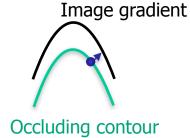






Given an initial pose estimate:

- Find the occluding contours.
- Find closest edge points in the normal direction.
- Re-estimate pose to minimize sum of square distances.
- Iterate until convergence.



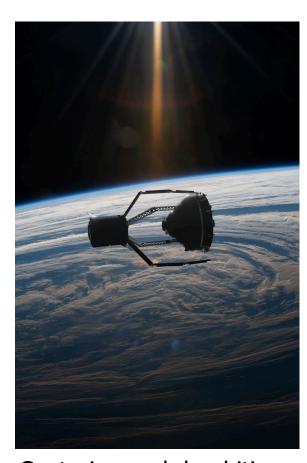


Visual Servoing

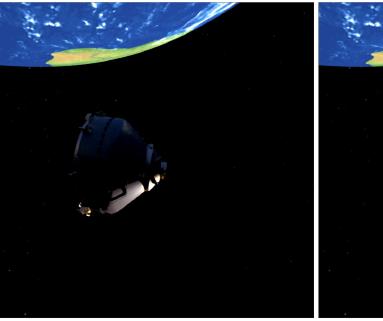


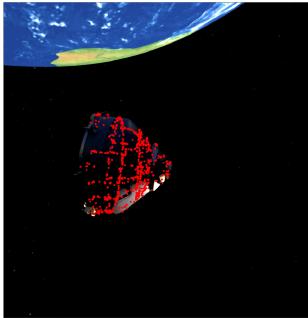


Space Cleaning



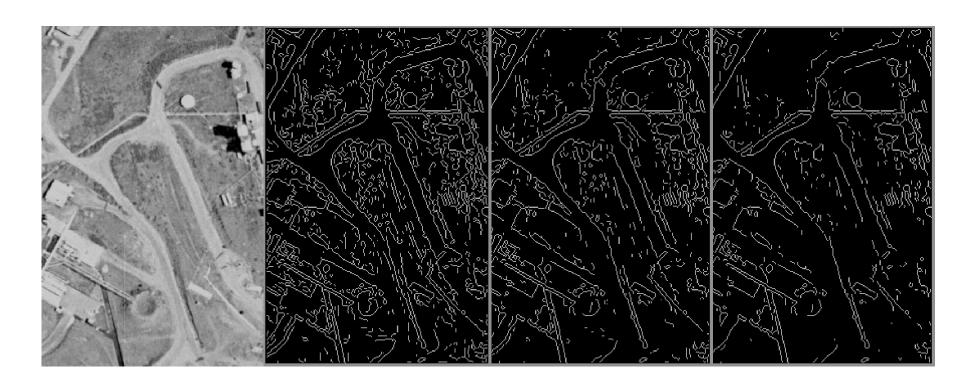
Capturing and deorbiting a dead satellite.





- A more sophisticated version of this old algorithm will blast off in 2025!
- ESA does not yet trust neural nets for such a mission.

Limitations of the Canny Algorithm

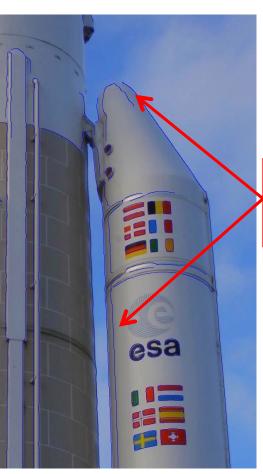


There is no ideal value of σ !



Steep Smooth Shading





- Rapidly varying gray levels.
- Large gradients.

→ Shading can produce spurious edges.



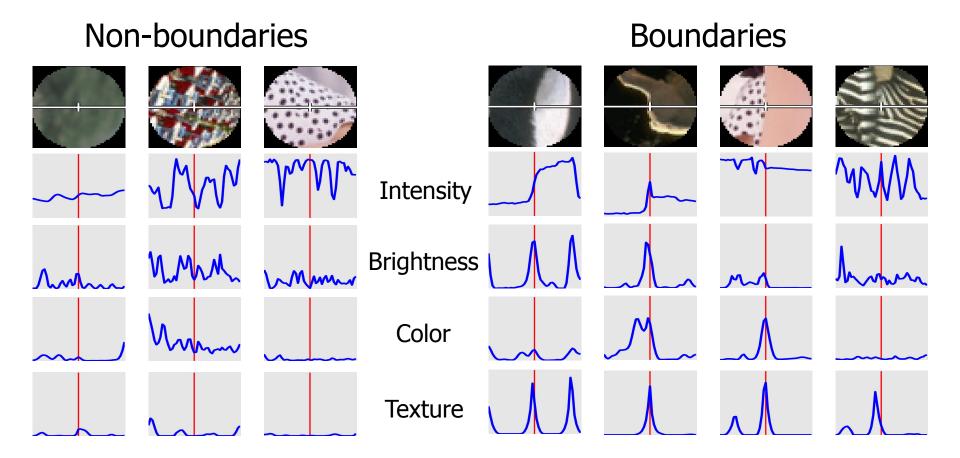
Texture Boundaries





- Not all image contours are characterized by strong contrast.
- Sometimes, textural changes are just as significant.

Different Boundary Types



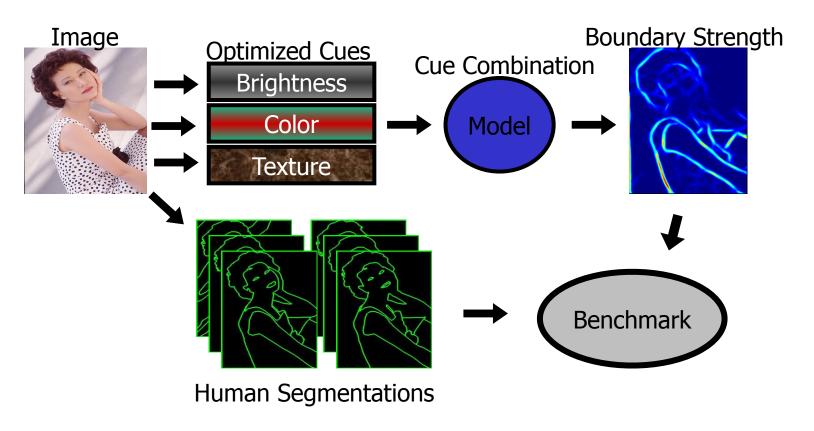


Training Database



1000 images with 5 to 10 segmentations each.

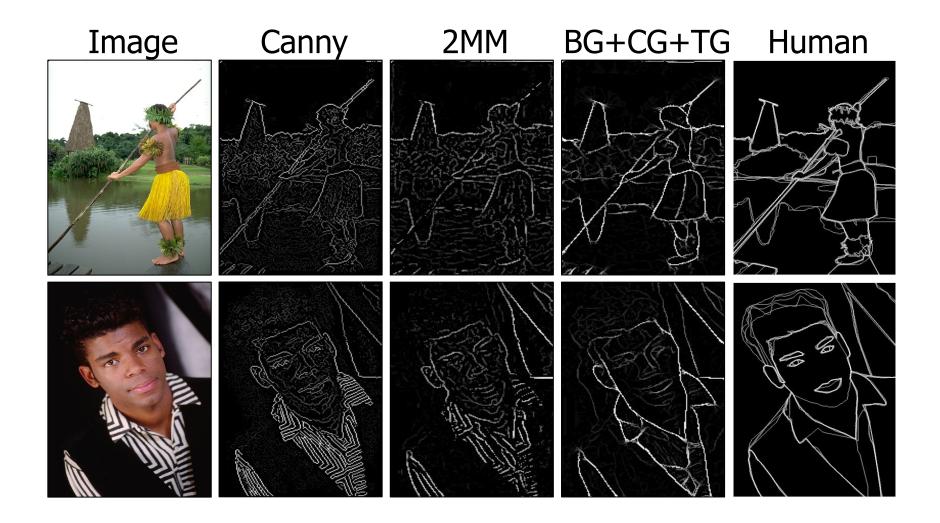
Machine Learning



Learn the probability of being a boundary pixel on the basis of a set of features.

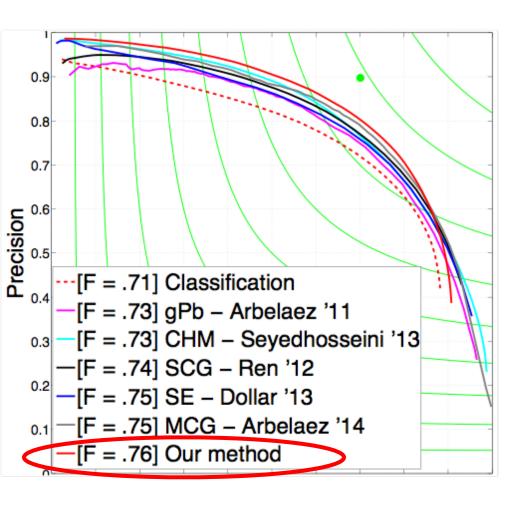


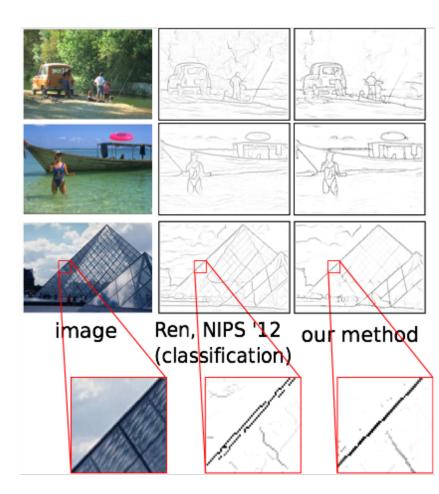
Comparative Results





Classification vs Regression

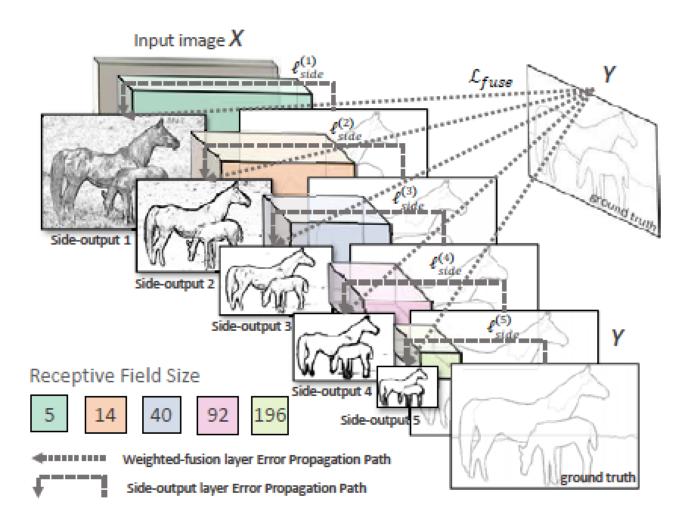




Yes!

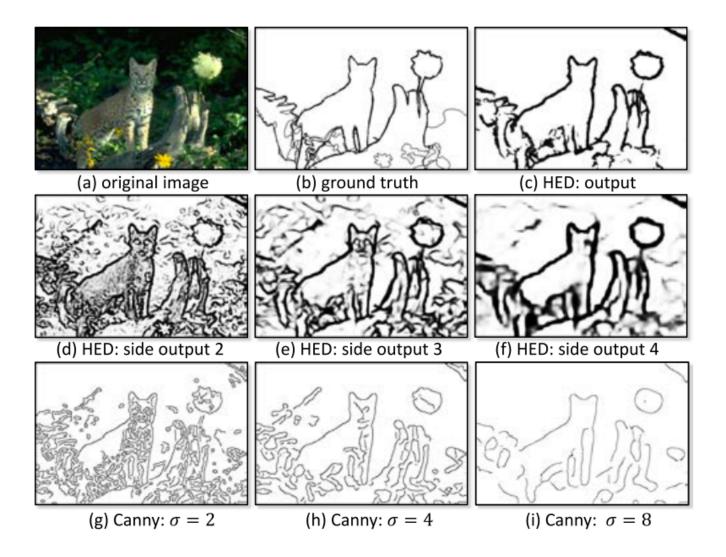


Deep Learning

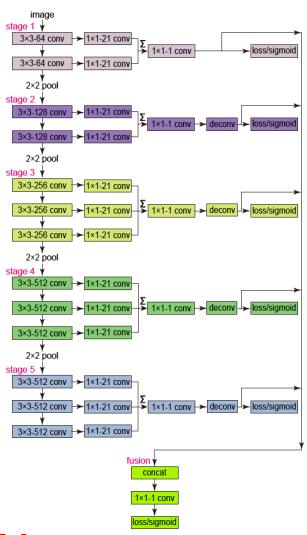


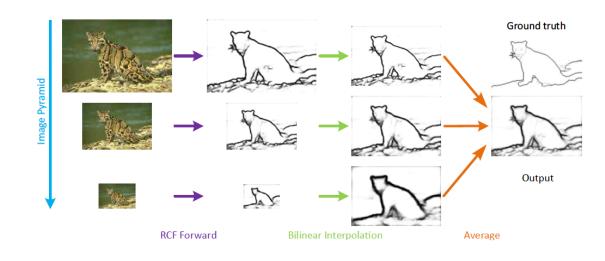


Deep Learning Vs Canny

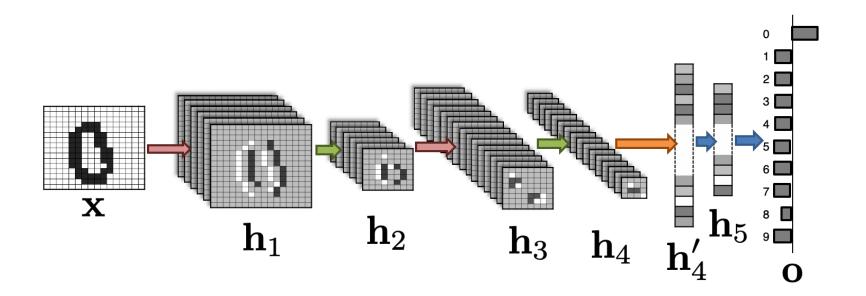


Deeper Learning





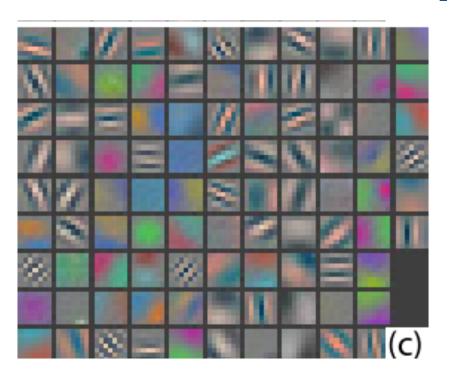
Convolutional Neural Network

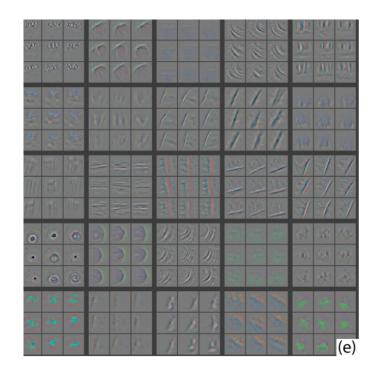


- Succession of convolutional and pooling layers.
- Fully connected layers at the end.
- —> Will be discussed in more detail in the next lecture.



A Partial Explanation?



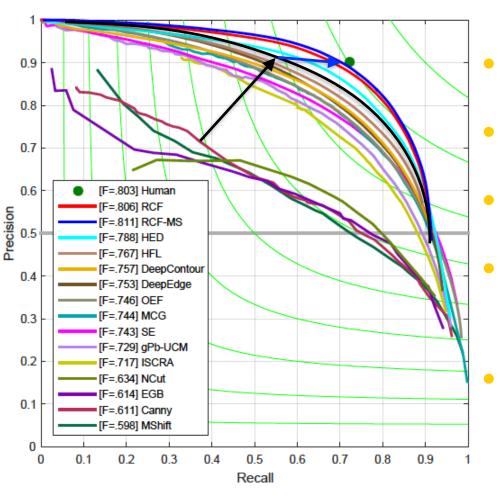


First and second layer features of a Convolutional Neural Net:

- They can be understood as performing multiscale filtering.
- The weights and thresholds are chosen by the optimization procedure.



50 Years Of Edge Detection



- Convolution operators respond to steep smooth shading.
- Parametric matchers tend to reject non ideal edges.
- Arbitrary thresholds and scale sizes are required.
- Learning-based methods need exhaustive databases.
 - There still is work to go from contours to objects.

Canny, PAMI'86 —> Sironi et al. PAMI'15

Sironi et al. PAMI'15 —> Liu et al., CVPR'17



Let us talk about deep networks.